Math 171A: Linear Programming

Lecture 1

Overview of the Class:
Introduction to Optimization

Elizabeth Wong  © 2019
http://math.ucsd.edu/~elwong/171a

Monday, January 7th, 2019

Overview of Math 171A

- Lecture slides are available for download on the class website
- Generally will be available the evening before lecture
- Will be updated after class
- If you print, please be mindful of the paper used – there are paper-friendly 4x4 versions of the slides available
- The login is your UCSD username and the password is your UCSD PID
- For example, 'elwong' and 'A01234567'

Overview of Math 171A

Instructor Office Hours
Elizabeth Wong  Mondays, Fridays 2p - 3:15p APM 5848

TAs Office Hours Thursday Section(s)
Xindong Tang  APM 6303  A01, A02 (8pm, 9pm in B402A)
Ziyan Zhu  APM 2202  A03, A04 (8pm, 9pm, in 2402)
Bingni Guo  APM 6442  A05 (7pm in B402A)

No discussions this week!

Class webpage: http://math.ucsd.edu/~elwong/171a

Overview of Math 171A

The grade is based on homework, 2 midterms and a final

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>20%</td>
</tr>
<tr>
<td>Midterm #1</td>
<td>20%</td>
</tr>
<tr>
<td>Midterm #2</td>
<td>20%</td>
</tr>
<tr>
<td>Final</td>
<td>40%</td>
</tr>
</tbody>
</table>

UC San Diego | Center for Computational Mathematics | Math 171A: Linear Programming Winter 2019
Overview of Math 171A

Homework

- Written part: submitted through www.gradescope.com
- Programming part: Jupyter notebooks on datahub.ucsd.edu (Julia!)
  - Some programming experience recommended (Matlab, Math 18/20D)
  - No experience with Julia/Jupyter required

This week:
- Expect an email from gradescope.com
- Check that you can login to datahub.ucsd.edu

What is Optimization?

*Optimization* is the search for the best solution to a problem under a given set of circumstances.

The *study* of optimization involves the formulation of a *mathematical model* of a given process or resource for which “optimizing” means minimizing or maximizing a function.

Topics for this quarter

- The formulation of simple linear programs
  - The graphical method
- Review of linear algebra
- Linear programs with constraints in all-inequality form and standard form
  - Optimality conditions
- The simplex method
- Finding a feasible point
- Primal and dual forms of a linear program
- Interior methods
A Simple Modeling Example

The JuiceCo company produces CranApple and AppleBerry juice. Both are made from a blend of apple and cranberry syrup. At the moment, JuiceCo has 200 quarts of cranberry juice and 100 quarts of apple juice. CranApple requires 1 quart of apple and 3 quarts of cranberry juice. AppleBerry requires 2 quarts of apple and 2 quarts of cranberry. CranApple sells for $3 per quart and AppleBerry for $4. How many quarts of CranApple and AppleBerry juice should JuiceCo produce to maximize profits?

Variables

Step 1: Identify the variables

\[ x = \text{quarts of cranapple juice produced} \]
\[ y = \text{quarts of appleberry juice produced} \]

The problem has 2 variables. We want to find the “best” or “optimal” values for \( x \) and \( y \)

Objective

Step 2: Identify the objective

Find \( x \) and \( y \) to maximize profit

CranApple sells for $3 per quart and AppleBerry for $4.

The objective function is \( 3x + 4y \)

Constraints

Step 3: Identify the constraints

What are some of the limitations on JuiceCo during production?

- JuiceCo has 200 quarts of cranberry juice and 100 quarts of apple juice.
- CranApple requires 1 quart of apple and 3 quarts of cranberry juice.
- AppleBerry requires 2 quarts of apple and 2 quarts of cranberry.

\[ x + 2y \leq 100 \text{ (total apple juice available)} \]
\[ 3x + 2y \leq 200 \text{ (total cranberry juice available)} \]

Additionally, we can’t produce negative amounts of cranapple or appleberry juice! \( \Rightarrow x, y \geq 0 \)
Mathematical problem

\[
\begin{align*}
\text{maximize} & \quad 3x + 4y \\
\text{subject to} & \quad x + 2y \leq 100 \\
 & \quad 3x + 2y \leq 200 \\
 & \quad x \geq 0 \\
 & \quad y \geq 0
\end{align*}
\]

Notice, the objective function and the constraints are all **linear**

Structure of the class

**Math 171A:** minimizing **linear** functions subject to **linear** constraints

**Linear programming**

**Math 171B:** minimizing **nonlinear** functions subject to **nonlinear** constraints

**Nonlinear programming**

Some examples of optimization problems

Data science:
- **Compressive sensing**: minimizing combinations of norms.
- Primal and dual forms of linear support-vector machines.

Inverse problems:
- Medical imaging
- Geophysical applications
  - Estimating the Earth’s electrical conductivity
  - Mineral and oil exploration
- Controlling the environment
  - Cleaning groundwater contaminants

Air travel:
- Flight attendants
  - A flight scheduling algorithm assigned the flight crew to your plane
- Aircraft design
  - An optimization algorithm was used to design shape of the plane
- Passenger routing
  - NBA game locations and dates are determined using optimization
Design technology:
- Yacht design
  - Optimal design of America’s Cup boats
- Space-X Falcon 9
  - The take-off and re-entry trajectories calculated to minimize structural stress and heating
  - Determination of the surface temperature using sensors
  - Minimization of fuel consumption and maximization of payload
- Calculation of trajectories for spacecraft
  - Optimal control of long-term interplanetary missions
  - Design of orbiters
- Control of unmanned autonomous vehicles (UAVs)

Can we use methods from calculus?

For example:

\[ \text{minimize } x^2 \]

The objective function \( f(x) = x^2 \) is minimized at \( x^* = 0 \)

The derivative is zero at \( x^* \)

\[ \frac{df}{dx} = f'(x) = 2x = 0 \]

Similarly,

\[ \text{minimize } x^2 + y^2 \]

The function \( f(x) = x^2 + y^2 \) is minimized at \( x^* = 0, y^* = 0 \).

The partial derivatives are zero at \( x^* \) and \( y^* \)

\[ \frac{\partial f}{\partial x} = 2x = 0 \]
\[ \frac{\partial f}{\partial y} = 2y = 0 \]
Our problem is more difficult because of the inequality constraints.

How do we solve this problem?

maximize \( 3x + 4y \)
subject to \( x + 2y \leq 100 \)
\( 3x + 2y \leq 200 \)
\( x \geq 0 \)
\( y \geq 0 \)
Problem Geometry

maximize \(3x + 4y\)
subject to
\(3x + 2y \leq 200\)
\(x + 2y \leq 100\)
\(x \geq 0\)
\(y \geq 0\)

Basic result of linear programming

The \textit{optimal solution} \((x^*, y^*)\) lies at a “corner-point” of the boundary of the feasible region.