Math 20D (Winter 2016)
Midterm Exam 1 – Version A
Instructor: Elizabeth Wong

Instructions:
1. Write your Name, Section, PID, and Exam Version on your blue book.
2. Read each question carefully. Justify all answers. No credit will be given for unsupported answers.
3. Write your solutions clearly
   (a) Start each problem (#1, #2, ...) at the top of a new page.
   (b) Carefully indicate the number and letter of the question you are answering.
4. No electronic devices allowed.

0. (Bonus 1 pt) Read and follow the instructions given above.

1. (10 pts) Find the solution to the initial value problem and determine the interval in which the solution is defined:

   \[ t^2 y' + 2ty = 3e^{3t}, \quad y(1) = 0 \]

2. Consider the following differential equation:

   \[ x^2y^3 - \sin x + (x^3y^2 + y) \frac{dy}{dx} = 0 \]

   (a) (2 pts) Show the equation is exact.
   (b) (8 pts) Find an implicit solution to the differential equation.

3. A tank contains 200 gallons of water with 40 kg of salt. Fresh water flows into the tank at a rate of 4 gallons per minute. The liquid in the tank is mixed completely so the concentration of salt is uniform. The tank is also leaking at a rate of 4 gallons per minute.

   (a) (4 pts) Set up the differential equation that gives the rate of change in salt in the tank at time \( t \).
   (b) (4 pts) Solve the differential equation you found in part (a).
   (c) (2 pts) What happens to the amount of salt in the long term (as \( t \to \infty \))?

4. Consider the following differential equation:

   \[ \frac{dy}{dt} = (9 - y^2)(y - 2) \]

   (a) (8 pts) Find all of the equilibrium solutions. Draw a phase line and determine whether each equilibrium solution is stable or unstable.
   (b) (2 pts) Suppose there is a solution \( y = \phi(t) \) to the differential equation that satisfies the initial condition \( y(0) = 4 \). What happens to \( \phi(t) \) as \( t \to \infty \)?