Instructions
1. No calculators or other electronic devices are allowed during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your Name, PID, and Section on the front of your Blue Book.
4. Write the Version of your exam on the front of your Blue Book.
5. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new page.
6. Read each question carefully, and answer each question completely.
7. Show all of your work; no credit will be given for unsupported answers.

1. (10 pts)
   (a) (4 pts) Determine where the solution of the following initial value problem is defined without solving for \( y(t) \).
   \[
   (t-2)y' + (\sin t)y = \frac{e^t}{t(t+3)}, \quad y(3) = 1
   \]
   (b) (6 pts) Solve the following initial value problem
   \[
   ty' + 2y = t^{-1}e^t, \quad t > 0, \quad y(1) = 0
   \]

2. (8 pts) Solve the differential equation
   \[
   \frac{dy}{dx} = -y^2(2 - \sin x)
   \]

3. (10 pts) Consider the differential equation
   \[
   (e^x + e^y + x^2)y' = -(e^y + xy^2)
   \]
   (a) (2 pts) Explain why the equation is exact.
   (b) (8 pts) Solve the differential equation (it suffices to give an implicit solution).

4. (8 pts) Consider the autonomous equation
   \[
   \frac{dy}{dt} = y^2 - 5y + 6
   \]
   (a) (3 pts) Find the equilibrium solutions.
   (b) (5 pts) Classify the equilibrium solutions as stable or unstable (justify your answer!).

5. (6 pts) Find the general solution to the following differential equations
   (a) (3 pts) \( y'' - 6y' + 9y = 0 \)
   (b) (3 pts) \( 4y'' + y = 0 \)

6. (8 pts) You have a small pool that holds 1000 gallons of water. Let \( Q(t) \) denote the grams (g) of chlorine in the pool at time \( t \) in minutes. Your pool man unfortunately did not take Math 20D and your pool is over-chlorinated. There are 450 grams of chlorine in the pool. To fix it, you add fresh water into your pool at a rate of \( r \) gal/min, and also drain the pool at the same rate.
   (a) (2 pts) Write down a differential equation and initial condition for this problem.
   (b) (4 pts) Solve the initial value problem for \( Q(t) \).
   (c) (2 pts) Find the rate \( r \) so that your pool will have 100 grams of chlorine in 2 hours.