First order equations

- Modelling problems
  - Concentration of salt in a water tank \((dQ/dt = \text{rate in} - \text{rate out})\)
- Linear equations: \(y' + p(t)y = g(t)\)
  - When and where is the existence and uniqueness of a solution guaranteed?
  - Method of integrating factors
    * Find \(\mu(t)\) so that \(\mu(t)y' + \mu(t)p(t)y = (\mu(t)y)'\); then \((\mu(t)y)' = \mu(t)g(t)\)
    * \(\mu(t) = e^{\int p(t)dt}\)
- Nonlinear equations:
  - Separable equations: \(M(x) + N(y)y' = 0\)
  - Exact equations: \(M(x, y) + N(x, y)y' = 0\)
    * Identify by checking whether \(M_y = N_x\)
    * Find a function \(\psi(x, y)\) such that \(\psi_x = M(x, y)\) and \(\psi_y = N(x, y)\); then \(\psi(x, y) = c\) defines an implicit solution
    * Using an integrating factor to make a differential equation exact
- Autonomous equations: \(dy/dt = f(y)\)
  - Finding equilibrium solutions; classifying whether the equilibrium solution is stable or unstable (phase lines, looking at behavior of \(f(y)\))