Final examination

- Tuesday December 8 from 3p – 6p in Center 101
- Bring a **photo ID**. We will be checking them during the exam.
- Bring a blue book (or 2 if you feel you need it).
  Also, you do NOT have to start each part of a problem on a new page, only each problem. For example, start problem #1 on a new page, #2 on a new page, etc...but NOT the individual parts (1a, 1b, 2a, etc...).
- You are allowed one 8”x11” sheet of handwritten notes (both sides).
- Check TED for your new seat assignment. (If you are left/right-handed and get put in a right/left-handed seat, you can move after everyone has been seated).
- No electronic devices allowed
What’s on the final?
Everything we’ve covered in class except Section 3.3. There will be some emphasis on material since the second midterm.

- Sections 1.1 – 1.5, 1.7 – 1.9
- Sections 2.1 – 2.3
- Sections 4.1 – 4.6
- Sections 3.1 – 3.2 (3.3 will not be on the final)
- Sections 5.1 – 5.3
- Sections 6.1 - 6.5

How long will it be?
8 - 10 problems, similar in format to those on the first and second midterm. You have 3 hours, but it should not take that long.
When are our office hours?

- Geoff: Friday and Sunday 5p-7p in APM 6446; Monday 2p-4p in Muir Woods
- Ben: Monday and Tuesday 12:30-2:30 in APM 5712
- Elizabeth: Sunday 11:30a - 1p, Monday 11a-3p in APM 5848
*Practice problems?*
Check our class webpage for problems and solutions. I’ve posted a few more problems and also listed a few problems from Section 6.5 earlier today (Friday).

(Section 6.5 is not part of the homework)

**What else?**
Midterm exams, previous practice problems, old homework problems.
Chapter 1

- Elementary row operations
- Row reduction – (reduced) echelon form
- Pivots
- Free variables
- Consistent/inconsistent systems
- Finding general solutions to a system of equations (homogeneous and non-homogeneous); parametric vector form
- Linear combinations, span
- Linear dependence, linear independence
- Linear transformations, one-to-one, onto
- Standard matrix for a linear transformation
Chapter 2

- Matrix operations and properties: addition, matrix multiplication, scalar multiplication, transpose
- Inverse matrix of a square matrix: definition and properties
- Invertible matrix theorem
Chapter 3

- Determinant of square matrix and properties
- Computing determinants by cofactor expansion
  - Triangular matrices
  - Matrices with many zeros in rows/columns
- Effect of row operations on determinant
Chapter 4

- Vector spaces and subspaces
  - Show something is a subspace of a vector space
- Dimensions of subspaces/vector spaces
- Basis for a vector space: what is it? how do you find it?
- Coordinates relative to a basis
- Nullspace, column space, row space of a matrix: what are they? how do you compute a basis for each?
- Rank Theorem
Chapter 5

- Characteristic polynomial
  * Eigenvalues and eigenvectors: what are they? how do you compute them for a given square matrix?
  * Eigenspaces; significance of the dimensions of the eigenspaces
  * Diagonalizing a matrix: know when can a matrix be diagonalized; know how to do it when it is possible
Chapter 6

- Dot/inner products, norms
- Orthogonality
- Orthogonal vectors; orthonormal vectors (orthogonal and normalized)
  - Orthogonal complement of a subspace
    - $\text{row}(A)^\perp = \text{null}(A)$
    - $\text{col}(A)^\perp = \text{null}(A^T)$
- Compute the orthogonal projection of a vector onto a subspace
- Distance between a vector and a subspace
  - Gram-Schmidt process
    - Finding an orthogonal/orthonormal basis for a subspace
    - $QR$ factorization, where $Q$ has orthonormal columns, $R$ is upper triangular
      - Compute $Q$ via Gram-Schmidt and normalizing
      - Compute $R = Q^T A$
- Matrices with orthonormal columns; properties
  - Solving a least-square problem (normal equations $A^T A x = A^T b$)
Invertible Matrix Theorem

Let \( A \) be an \( n \times n \) matrix and define the linear transformation \( T(x) = Ax \).

1. \( A \) is invertible
2. \( A \) is row equivalent to the \( n \times n \) identity matrix \( I_n \)
3. \( A \) has \( n \) pivots
4. \( Ax = 0 \) has only the trivial solution
5. The columns of \( A \) form a linearly independent set
6. \( T(x) \) is one-to-one
7. \( Ax = b \) has a unique solution for each \( b \in \mathbb{R}^m \)
8. The columns of \( A \) span \( \mathbb{R}^n \)
9. \( T(x) \) maps \( \mathbb{R}^n \) onto \( \mathbb{R}^n \)
10. There is an \( n \times n \) matrix \( C \) such that \( CA = I \)
11. There is an \( n \times n \) matrix \( B \) such that \( AB = I \)
12. \( A^T \) is invertible
13. The columns of \( A \) form a basis for \( \mathbb{R}^n \)
14. \( \text{col}(A) = \mathbb{R}^n \iff \dim(\text{col}(A)) = n \iff \text{rank } A = n \)
15. \( \text{null}(A) = \{ \vec{0} \} \iff \dim(\text{null}(A)) = 0 \)
16. \( \det(A) \neq 0 \)
17. 0 is not an eigenvalue of \( A \)