Math 20F – Fall 2015 (Lecture C)  
Midterm 1 Exam – Version A  
Instructor: E. Wong – October 21, 2015

Instructions:

1. Write your Name, Section, and PID on the front of your blue book.
2. Write the Version of your exam on the front of your blue book.
3. Read each question carefully. Justify your answers. No credit will be given for unsupported answers.
4. Write your solutions clearly:
   (a) Carefully indicate the number and letter of the question you are answering.
   (b) Start each question on a new side of a page.
5. No electronic devices are allowed.
6. You may use your handwritten (8.5”x11”) sheet of notes, but nothing else during this exam.
0. (2 pts) Read and follow the instructions on the other side of this page.

1. (12 pts) Consider the following matrix:

\[ A = \begin{bmatrix}
4 & 1 & 0 \\
2 & -1 & 3 \\
0 & 5 & -10 \\
2 & 0 & 1
\end{bmatrix}. \]

(a) (4 pts) Is \( A\vec{x} = \vec{b} \) consistent for all \( \vec{b} \in \mathbb{R}^4 \)?

(b) (5 pts) Find the solution set of the homogeneous equation defined by \( A \).

(c) (3 pts) Are the columns of \( A \) linearly independent or dependent? If dependent, find a linear dependence relation.

2. (10 pts) Consider the transformation \( T(\vec{x}) = A\vec{x} \) and the vector \( \vec{b} \), where

\[ A = \begin{bmatrix}
1 & -1 & 0 & 3 \\
0 & 3 & 0 & -1 \\
0 & 0 & 0 & -2
\end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix}
1 \\
7 \\
-4
\end{bmatrix}. \]

(a) (2 pts) What is the domain and codomain of \( T \)?

(b) (4 pts) Is \( T(\vec{x}) \) one-to-one?

(c) (4 pts) Is \( \vec{b} \) in the range of \( T \)?

3. (4 pts) Consider the following vectors:

\[ \vec{v}_1 = \begin{bmatrix}
-1 \\
0
\end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix}
1 \\
3
\end{bmatrix} \]

(a) (2 pts) Find two vectors (not \( \vec{v}_1 \) or \( \vec{v}_2 \)) that are in the span of \( \{ \vec{v}_1, \vec{v}_2 \} \).

(b) (2 pts) Suppose \( T \) is a linear transformation such that:

\[ T(\vec{x}) : \mathbb{R}^2 \to \mathbb{R}^2 \quad \text{with} \quad T(\vec{v}_1) = \vec{y} \quad \text{and} \quad T(\vec{v}_2) = \vec{z}. \]

What does \( T \) map \( 2\vec{v}_1 + \vec{v}_2 \) to?

4. (4 pts) Consider the transformation \( T(\vec{x}) : \mathbb{R}^2 \to \mathbb{R}^3 \) defined as:

\[ T(\vec{x}) = \begin{bmatrix}
2x_1 - 5x_2 \\
x_1 + 2x_2 \\
x_2
\end{bmatrix}. \]

Find the standard matrix of this transformation.

5. (4 pts) Let \( A \) be a \( 3 \times 4 \) matrix, \( B \) be a \( 4 \times 2 \) matrix, and \( C \) be a \( 4 \times 4 \) matrix. Provide the dimensions of the following matrices or state that the matrix is not defined.

(a) \( AB \) \quad (b) \( B^T C \) \quad (c) \( A^T C \) \quad (d) \( B^2 \)