Instructions:

1. Write your Name, Section, and PID on the front of your blue book.
2. Write the Version of your exam on the front of your blue book.
3. Read each question carefully. Justify your answers. No credit will be given for unsupported answers.
4. Write your solutions clearly:
   (a) Carefully indicate the number and letter of the question you are answering.
   (b) Start each question on a new side of a page.
5. No electronic devices are allowed.
6. You may use your handwritten (8.5”x11”) sheet of notes, but nothing else during this exam.

Section Info:

- C01: Thu 4 – 5 Ben
- C02: Thu 5 – 6 Ben
- C03: Thu 6 – 7 Ben
- C04: Thu 7 – 8 Ben
- C05: Thu 8 – 9 Geoff
- C06: Thu 4 – 5 Geoff
- C07: Thu 5 – 6 Geoff
- C08: Thu 6 – 7 Geoff
0. (2 pts) Read and follow the instructions on the other side of this page.

1. (9 pts) Consider the following matrix: 
\[ A = \begin{bmatrix} 1 & 0 & -1 & 2 & 5 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \]

(a) (6 pts) Show that \( \text{null}(A) \) is a subspace of \( \mathbb{R}^5 \).
(b) (3 pts) Find a basis for \( \text{null}(A) \).

2. (4 pts)
(a) (1 pt) State the Rank Theorem for an \( m \times n \) matrix \( A \).
(b) (3 pts) Suppose \( A \) is \( 18 \times 23 \). What must the dimension of \( \text{null}(A) \) be for \( Ax = b \) to have a solution for all vectors \( b \)?

3. (9 pts) The matrix \( A \) is row equivalent to the matrix \( B \):
\[ A = \begin{bmatrix} 4 & 0 & -1 & 2 \\ 4 & 2 & -7 & 2 \\ 0 & 2 & -6 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 0 & -1 & 2 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \]

(a) (2 pts) What is the rank of \( A \)? What is the rank of \( A^T \)?
(b) (2 pts) Find a basis for \( \text{col}(A) \).
(c) (2 pts) Find a basis for \( \text{row}(A) \).
(d) (1 pt) Find a basis for \( \text{col}(A^T) \).
(e) (2 pts) What is the dimension of \( \text{null}(A^T) \)?

4. (4 pts) The set \( B = \{ \vec{b}_1, \vec{b}_2 \} \), where \( \vec{b}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \) and \( \vec{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \) is a basis for \( \mathbb{R}^2 \).

(a) (2 pts) Given the coordinate vector \( [x]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \), find \( x \), the coordinates of \( [x]_B \) relative to the standard basis.
(b) (2 pts) Let \( \vec{y} = \begin{bmatrix} 2 \\ 9 \end{bmatrix} \). Find \( [y]_B \), the coordinates of \( \vec{y} \) relative to the basis \( B \).

5. (8 pts)
(a) (4 pts) Compute the determinants of \( A \) and \( B \):
\[ A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ -3 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 0 & 4 & -2 & 0 \\ 0 & 1 & -2 & -4 \end{bmatrix} . \]

(b) Suppose \( C \) and \( D \) are an \( n \times n \) matrices with \( \det(C) = -5 \) and \( \det(D) = 2 \). Compute the following determinants.
   i. (2 pts) \( \det(C^{-1}D) \)
   ii. (2 pts) \( \det(C^T D^2) \)