Name: **PID:**
Discussion Section - No: **Time:** **TA's name:**

Math 10C - Final (Lecture A, Winter 2007)

Duration: 3 hours
Please close your books, turn your calculators off and put them away.
You can use one page of notes.
To get full credit you should support your answers.

1.
a) (3 points) Find the Taylor polynomial of degree 3 for e^x about $x = 0$. Use this Taylor polynomial to approximate the Euler constant e.

b) (3 points) Find the sum of the convergent series

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

#	Score
1	
2	
3	
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8	
Total	

2. Describe in words the graphs of the equations in 3-space in all parts.

a) (2 points) $x = y$

b) (2 points) $x = y = z$

c) (2 points) $z = \sqrt{1 - x^2 - y^2}$

3. Let

- $p = (0, 1, 0)$, $q = (-1, 1, 2)$, $r = (2, 1, -1)$ be points in 3-space,
- $\overrightarrow{pq}$, $\overrightarrow{pr}$ be the displacement vectors from p to q and from p to r, respectively,
- $\vec{n}$ be a normal vector to the plane containing p, q and r.

You don't need to calculate $\vec{n}$ to solve any of the parts.

a) (2 points) Using the definition of a normal vector simplify $\overrightarrow{pq}.\vec{n}$

b) (2 points) Explain how you can obtain the normal vector $\vec{n}$ using the displacement vectors $\overrightarrow{pq}$ and $\overrightarrow{pr}$.

c) (2 points) Find the displacement vectors $\overrightarrow{pq}$ and $\overrightarrow{pr}$.

d) (2 points) Find the angle between $\overrightarrow{pq}$ and $\overrightarrow{pr}$.

e) (2 points) Find the area of the triangle with vertices p, q and r.

4.

a) (4 points) Let

$$g(x,y) = f(u(x,y), w(x,y)), \quad u(x,y) = x^2 y \text{ and } w(x,y) = x + 2y.$$

Express the gradient vector $\nabla g(x,y)$ for any x and y in terms of the partial derivatives $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial w}$.

b) (2 points) Let

$$h(x,y) = e^{2x+y}(x+y).$$

Find the derivative $\frac{\partial^2 h}{\partial x \partial y}$.

c) (2 points) Let

$$s(x,y) = e^x \ln y.$$

Find the directional derivative $s_{\vec{v}}(2,3)$ in the direction of the unit vector $\vec{v} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$.

5. The plane tangent to $f(x, y)$ about $x = 3$ and $y = 1$ is given by

$$L(x, y) = z = 3 - 2x + 4y$$

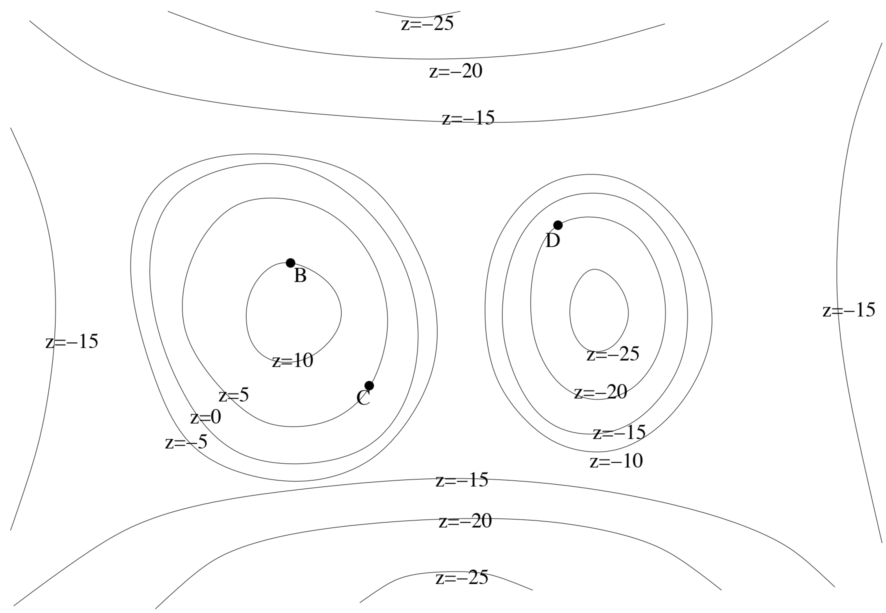
a) (2 points) Find $f(3, 1)$.

b) (2 points) Find the partial derivatives $f_x(3, 1)$, $f_y(3, 1)$.

c) (2 points) Find a normal vector to the tangent plane $L(x, y)$.

d) (2 points) Find two vectors (not parallel or opposite to each other) contained on the tangent plane $L(x, y)$.

6. Answer all of the parts for the function whose contour diagram is illustrated below.

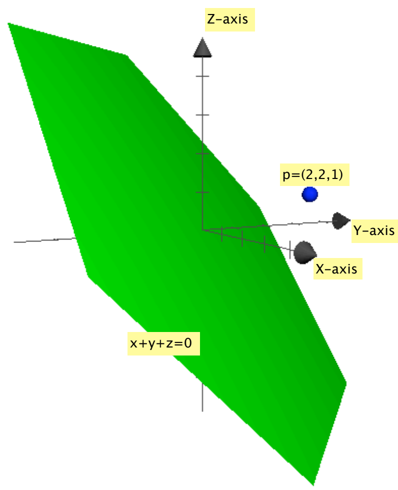


a) (2 points) Among the points B, C and D determine where the function is steepest. Explain your reasoning.

b) (2 points) Draw vectors on the contour diagram at C and D pointing in the direction of the gradient vectors. Explain in words also how the gradients vectors are aligned with respect to the contours.

c) (2 points) Mark three points on the contour diagram that are possibly a local minimum, a local maximum and a saddle point with the letters M, X and S, respectively.

7. (6 points) Find the unique point on the plane $x + y + z = 0$ that is closest to the point $p = (2, 2, 1)$ in 3-space by setting up an (unconstrained) optimization problem depending on two variables. The function that needs to be minimized is the distance from the point p to a point p_1 on the plane. The closest point should be a critical point of the optimization problem.



8. In this question we want to minimize the surface area of a box whose base is a square and volume is $8000cm^3$.

a) (2 points) Find a constrained minimization problem over two variables whose solution gives the minimum surface area possible.

b) (2 points) Find the Lagrangian function associated with the constrained optimization problem in part **a)**.

c) (2 points) By finding the critical points of the Lagrangian function determine the dimensions of the box that yield the smallest surface area.