Preface

There is no question that Paul Erdős must be counted among the mathematical giants of the 20th century. His fundamental discoveries and profound contributions in so many areas of mathematics form a record which may never again be matched. However, there is one area in which Paul surpassed everyone else by a large margin. This was his ability to formulate problems. And it wasn't just the quantity of problems which was so unbelievable (they numbered in the thousands). But rather, it was their quality. Paul had the uncanny ability time after time to identify a fundamental roadblock in some particular line of approach, and to capture it in a well-chosen (often innocent-looking) problem, one which could seem to be just within reach if you could only stretch out a little more than you had before. So often the new insights generated by solving such problems led to new tools and techniques for subsequently making substantial advances in the area under investigation. That Paul was able to do this so consistently over his entire lifetime is a mark of his true genius.

One might wonder why Erdős was as focused as he was on formulating and solving problems in his very many areas of mathematics. This is probably best understood by hearing Paul's own explanation, taken from the last paper[1] on problems he ever wrote, shortly before his death:

"Problems have always been an essential part of my mathematical life. A well chosen problem can isolate an essential difficulty in a particular area, serving as a benchmark against which progress in this area can be measured. An innocent looking problem often gives no hint as to its true nature. It might be like a 'marshmallow,' serving as a tasty tidbit supplying a few moments of fleeting enjoyment. Or it might be like an 'acorn', requiring deep and subtle new insights from which a mighty oak can develop. As an illustration of how hard it can be to judge the difficulty of a problem, I'd like to tell the following anecdote concerning the great mathematician David Hilbert. Hilbert lectured in the early 1930's on problems in mathematics and said something like this — probably all of us will see the proof of the Riemann Hypothesis, some of us (but probably not I) will see a proof of Fermat's Last Theorem, but none of us will see the proof that $2^{\sqrt{2}}$ *is transcendental. In the audience was Carl Ludwig Siegel, whose deep research contributed decisively to the proof by Kusmin a few years later of the transcendence of* $2^{\sqrt{2}}$*. In fact, shortly thereafter Gelfand and a few weeks later Schneider independently proved that* α^β *is transcendental if* α *and* β *are algebraic,* β *is irrational and* α *is not equal to* 0 *or* 1.

[1] P. Erdős, Some of my favorite problems and results, *The Mathematics of Paul Erdős, I* (R. L. Graham and J. Nešetřil, eds.), 47-67, Springer-Verlag, Berlin, 1996.

In this note I would like to describe a variety of my problems which I would classify as my favorites. Of course, I can't guarantee that they are all "acorns", but because many have thwarted the efforts of the best mathematicians for many decades (and have often acquired a cash reward for their solutions), it may indicate that new ideas will be needed, which can in turn, lead to more general results, and naturally, to further new problems. In this way, the cycle of life in mathematics continues forever."

Paul Erdős has been described as "the prince of problem solvers and the absolute monarch of problem posers". We hope that this book will help to serve as a living testament to this well-deserved description.

Fan and Ron
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