As a mathematician, Ron Graham is certainly no slouch. He’s in the Guinness Book of World Records for having used the largest number in a mathematical proof, a number so incomprehensibly large that it can only be expressed with a special esoteric notation, in which exponents are heaped on exponents to form a staggering tower of digits. As an athlete, Graham is no lightweight either. He has bowled two 300 games, is vicious at racket sports, and is quite at home on a trampoline. He can juggle six balls and is a past president of the International Jugglers Association. He has always managed to combine mental gymnastics with physical ones: he put himself through college as a circus acrobat, and he had one of his greatest mathematical insights while performing a back somersault with a triple twist. And so DISCOVER’s interest was piqued when he invited us to Cirque du Soleil and promised to introduce us to the mathematics and biomechanics of circus acts. What we learned that night is captured in the photo-story “Circus Science,” on page 56.
For centuries mathematicians have had a special fondness for juggling (the tenth-century mathematician Abu Sahl liked to juggle glass bottles in a Baghdad marketplace), but in the past decade they’ve taken their affection to a new level—they’ve invented juggling math. One of the founders of this fusion is Ron Graham (shown here), a mathematician at AT&T Bell Labs and a former president of the International Jugglers Association.

Any juggling routine is made up of a repeating pattern of hand-to-hand tosses, each of which may take a different amount of time. Graham and his colleagues represent such a pattern with a string of numbers that indicate how long a ball is in the air: a quick pass from one hand to the other is a 1, a high arc may be a 5. Thus, a three-ball shower—in which a juggler passes each ball from hand to hand (111) and then throws it high in the air (555) so that the balls seem to be tracing out a big circle—is 15.

But juggling math is more than notation. Graham discovered that given some simple assumptions—such as, you can’t catch two balls with one hand at the same time—he could solve equations that revealed things about both juggling and mathematics. The average of the numbers in any sequence, for example, equals the number of balls needed to perform the corresponding routine. Not all sequences represent performable juggling routines, and with his equations, Graham can test whether they are legitimate or not. (By punching the numbers into a computer, he can even see what a routine will look like.)

Juggling math, Graham has found, is full of unexpectedly elegant laws. For instance, how many different sequences are there that consist of four throws with fewer than five balls? The answer is simply $5^4$, or 625. “That’s a really nice number,” says Graham. “The question is, what is going on? Often, with a really nice answer, you suspect that something nice is going on more broadly.” Graham is now trying to find general laws for such sequences. “It’s all fitting into a general matrix, like a periodic table,” he says. “These things are finding their place.”