Nomination for Ronald L. Graham for AMS President

Written by *Gian-Carlo Rota*

Appeared in *Notices of AMS, 1991*

It is a privilege, as well as a singular honor, for me to place Ronald L. Graham's name in nomination for the Presidency of the American Mathematical Society. Graham is one of the charismatic figures in contemporary mathematics, as well as the leading problem-solver of his generation. For the last twenty-five years, he has been the central figure in the development of discrete mathematics. His seminal work has led to the birth of at least three new branches of mathematics: Ramsey theory, computational geometry, and worst case analysis of multiprocessing algorithms (now sometimes referred to as "Graham type analysis.")

Graham's characteristic quality is an indefatigable activity, both in the cause of mathematics, and on behalf of it applications.

Ron will never turn down a telephone call from a colleague, near or far, asking for help on a problem. Every one of his collaborators knows that Ron will somehow find whatever hours or days are needed to come up with some substantial suggestion, and frequently with the crucial step towards the solution. He is unusual, unique perhaps, in being able to effectively work on several problems at once, while carrying a full load of administrative work at Bell Labs.

Ron's positive view of mathematics and of science, as well as his entertaining lectures, have inspired generations of mathematicians.

Graham's first papers were in number theory, in fact his very first paper (1964) deals with a question which goes back to the Rhind Papyrus. It was known to the Egyptians that any "reasonably-sized" positive rational number can be represented as the sum of distinct unit fractions. A number of results were proved generalizing this fact; for example, Breusch and Stewart in 1954 independently that if $p/q > 0$ and q is odd, then p/q is the sum of a finite number of reciprocals of distinct odd integers. Graham proved the widest and deepest generalization this result. He have a simple necessary and sufficient condition for a rational number to be expressible as the finite sum of reciprocals of distinct positive integers taken from a preassigned set. A typical case of his theorem runs as follows: a rational number p/q can be expressed as a finite sum of reciprocals of distinct squares of integers if and only if

$$p/q \epsilon [0, \pi^2/6 - 1) \cup [1, \pi^2/6)$$

It was my good fortune to meet Ron Graham in 1965, a few years after he had received his Ph.D., and to pose to him a question which I believed to lie beyond the reaches of all techniques of the time. This was the geometric analog of Ramsey's theorem for vector spaces over finite fields. Let $GF(q)$ be a finite field with q elements. For every choice of p, t, n, there exists $N = N(p, t, n)$ such that for every t-coloring of the set of p-dimensional subspaces of an N-dimensional vector space V over $GF(q)$, there exists an n-dimensional subspace U such that all p-dimensional subspaces of U are monochromatic. In the limiting case of a "field with one element" one recovers Ramsey's theorem (1930). Graham introduced the notion of an "n-parameter set" (a kind of combinatorial geometry), in which a general form of a Ramsey-type could be proved which implied not only the classical theorem of Ramsey and its conjectured vector-space analog, by also van der Waerden's theorem on arithmetic progressions, and Hales and Jewett's theory of positional games.

Graham and Rothschild's achievement quickly attracted the attention of mathematicians in several countries and led to the development of what is now known as "Ramsey theory" (roughly speaking, the study of properties the must necessarily hold in a "sufficiently large" structure.) At several crucial stages, Graham's intervention played a decisive role; for example Graham (1973) initiated what is now called Euclidean Ramsey theory, that is, the proof of existence of Ramsey-type properties invariant under the Euclidean group. Ramsey theory has spilled over into set theory (logic and topological Ramsey theory) and even to dynamical systems.

Graham's work sheds light on a mysterious probabilistic method introduced by Erdős for proving existence proofs. No systematic procedure is known for turning Erdős's method into an explicit construction. Graham and Spencer (1971) were the first to provide an example to the contrary. They constructed a tournament such that for any set S of k vertices there is a vertex in the tournament which dominates all k elements of S. Similarly in collaboration with Diaconis and Morrison (1989), Graham computed the cutoff phenomenon in a random walk on an n-dimensional cube. This was the first Markov chain for which such a cutoff in random walks was explicitly determined.

Graham's later work in probability has led him to the invention of the concept of quasi-randomness. The idea is to deepen the known fact that a sufficiently "random" combinatorial object, for example, a graph, will almost surely have certain properties. Graham (in collaboration with F. Chung and R. Wilson, 1988) proves instead that certain properties for a "quasi-random" equivalence class: any family of objects having any one of such property will also have the other properties. This insight makes it possible to explicitly construct families of, say, graphs, which behave for all practical purposes like random graphs.

Graham's work in geometry is no less brilliant. In a celebrated result (obtained in 1975) Graham settled a 50-year old problem of H. Lenz, of determining the largest area of plane hexagon of unit diameter can have. He showed that such a hexagon is unique and has an area exceeding that of a regular hexagon of unit diameter by about 4%.

An important problem is that of efficiently locating an internal point of a given convex set. Early algorithms required $O(n^2)$ time and were thus rather inefficient. Graham (1972) observed that, in order to determine whether a point lies in a triangle defined by a set of N points, it is not necessary to test all such triangles. By performing a preliminary sorting step, Graham was able to invent an algorithm for finding such a point in linear time. He proved that the convex hull of N points in the plane can be found in $O(N \log N)$ time and $O(N)$ space, using only arithmetic operations and comparison. This is the first $O(N \log N)$ time algorithm to be discovered for the problem, and ten years later it was proved to be optimal. His method has blossomed into the field that is now called computational geometry.

Graham was quick to recognize the importance of NP-completeness. He showed (1977) that the classical problem of computing Steiner minimal trees for general planar point sets (on which several scientists in the nineteenth century, including Maxwell, had worked) is NP-complete. He was the first to establish a precise bound on worst-case performance of a combinatorial algorithm (1966). Again, his idea has now borne fruit in hundreds of papers applying it to such problems as scheduling and bin-packing, and in fact, in what is now called complexity theory.

My favorite theorem of Graham's is the following packing inequality (1969). Let K be a simplicial complex in the plane such that any two vertices are at a distance at least one apart from each other. Let $\alpha_0(K)$, $A(K)$, $P(K)$ and $\chi(K)$ be the number of vertices, the area, the perimeter, and the Euler characteristic of K. Then the following sharp inequality holds:

$$\alpha_0(K)/leq2A(K)/\sqrt(3)+P(K)/2+\chi(K).$$

Ronald L. Graham is one of the few mathematicians whose influence and leadership are acknowledged and appreciated in the scientific community at large, as well as among mathematicians. His mathematical depth, his broad vision, as well as his effectiveness in management make him, in my opinion, the candidate who will successfully steer the Society through the difficult years ahead.