CSE 202 Final Take-Home Exam, Winter 2008

Instructions: The exam should be handed in before 3pm, Thursday March 20. Professor Graham will be at EBU3 2136 between 2-3 that Thursday. If you wish to hand in earlier, just give it to the professor or our TA before the deadline.

Before giving more detailed descriptions of algorithms, describe in a brief (1-3 sentence) paragraph the main ideas and techniques. Complete pseudo-code is not necessary as long as you clearly specify how the algorithm works. You may use without proof any well-known algorithms and lower bounds for standard problems, as long as you state precisely and correctly the known result.

Give at least an informal proof for all answers; for algorithms, this should include some convincing argument of correctness, and a time analysis.

1. (25 pts)
Suppose we have a 2-dimensional table $A(i, j)$ such that all rows and columns are sorted, i.e., for all $1 \leq i \leq i' \leq n$ and $1 \leq j \leq n$, we have

$$A(i, j) \leq A(i', j) \quad \text{and} \quad A(j, i) \leq A(j, i').$$

Give an efficient algorithm which, for an input value $v$, finds an entry $(i, j)$ such that $A(i, j) = v$, if one exists. Can you beat a linear algorithm (in $n$)?

2. (25 pts)
Suppose that there is a network of roads $G = (V, E)$ connecting a set of $n$ cities. Each road $e$ in $E$ is associated with a length $l_e > 0$. There two major cities $A$ and $B$. There is a proposal to add one new highway (as an edge) to this network and there is a list of $k$ of potential highways which can be built. As a designer for the transportation department you are asked to choose the road whose addition to the existing network $G$ would result in the maximum decrease in the driving distance between $A$ and $B$. Give an efficient algorithm for solving this problem.

3. (25 pts)
A directed graph $G = (V, E)$ is said to be acyclic if it does not contain any directed cycle. Give an efficient algorithm to find a longest directed path in an acyclic graph $G$.

4. (25 pts)
Suppose we wish to assign $n$ students to $m$ tutors satisfying the following conditions: There are $k$ distinct courses $C_1, C_2, \ldots, C_k$. Each student identifies the subset of courses that he/she is interested in. Each Tutor identifies the subset of courses that he/she is specialized.

Give an efficient algorithm for assigning students to tutors so that each student is assigned to one tutor, each tutor gets at least 1 but at most 3 students who are interested in courses that the tutor specializes, if this is possible.

5. (25 pts)
Suppose we have two processors and $m$ jobs that require time $t_1, t_2, \ldots, t_k$. Give a greedy algorithm to minimize the makespan and provide a worst-case analysis. For example, can you show that the makespan of your greedy algorithm is no more than $6/5$ times the makespan of the optimal algorithm? Can you show that the makespan of your greedy algorithm is no more than $7/6$ times the makespan of the optimal algorithm?