E^n = Euclidean space

S^{n-1} = unit sphere in E^n

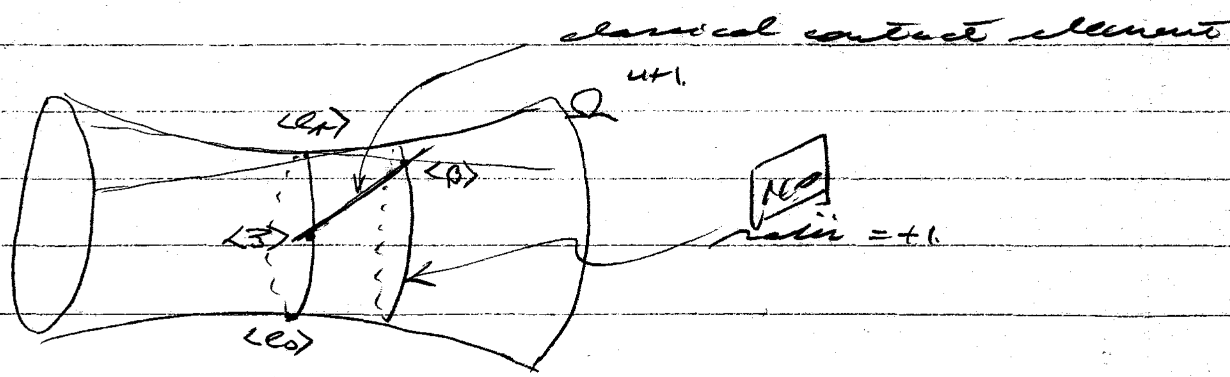
$$\mathfrak{Z}(x) = \begin{pmatrix} 1 \\ x_1 \\ \vdots \\ x_n \\ 0 \\ xx \end{pmatrix} \qquad \beta(x,b) = \begin{pmatrix} 0 \\ b_1 \\ \vdots \\ b_n \\ 1 \\ 2bx \end{pmatrix}$$

$0\bar{x}'\cdot\bar{x}' - 2\bar{b}\cdot\bar{x}' + 2\bar{b}\cdot\bar{x} = 0$

$\alpha^0\bar{x}'\cdot\bar{x}' - 2\bar{\alpha}\cdot\bar{x}' + \alpha^\infty = 0$

$xx = \bar{x}\cdot\bar{x}$

$bx = \bar{b}\cdot\bar{x}$ $\qquad bb = 1$



$\langle\mathfrak{Z}\rangle \in \Omega^{n+1} \cap \langle e_\infty\rangle^{+} = \Sigma^n$ $\qquad$ points

$\langle\beta\rangle \in \Omega^{n+1} \cap \langle e_0\rangle^{+} \cap \{\mathfrak{Z}^{\Gamma} = 1\}$ $\qquad$ hyperplane.

$(\mathfrak{Z}\,|\,\beta) = 0$, $\quad \langle\mathfrak{Z},\beta\rangle \subset \Omega^{n+1}$ $\quad$ } point and hyperplane are incident.

Contact form:

$$(\beta\,|\,d\mathfrak{Z}) = \left(\begin{pmatrix} 0 \\ \vec{b} \\ 1 \\ 2bx \end{pmatrix}, \begin{pmatrix} 0 \\ d\vec{x} \\ 0 \\ 2x\,dx \end{pmatrix} \right) = \bar{b}\cdot d\bar{x} = b_1\,dx_1 + \cdots + b_n\,dx_n$$

$\bar{b}\cdot\bar{b} = 1$ $\quad$ gives $\quad$ $\bar{b}\cdot d\bar{b} = 0$.

$\bar{\sigma}\cdot d\vec{x}$ contact form

$$\begin{array}{ccc} E^n \times S^{n-1} & \longrightarrow & \Lambda^{2n-1} \\ \Big\downarrow \text{proj} & & \Big\downarrow \\ E^n & \longrightarrow & \Psi^n \end{array}$$

$$\begin{array}{ccc} (x,\sigma) & \longrightarrow & \langle \xi(x), \beta(x,\sigma)\rangle \;\; \ell \\ & & \Big\downarrow \\ & & \ell \cap \langle e_r\rangle^{\perp} \\ x & \longrightarrow & \langle \xi(x)\rangle \end{array}$$

Aside: $E^n \times S^{n-1} \xrightarrow{\text{2 to 1}} E^n \times P(E^{n*})$

oriented contact elements

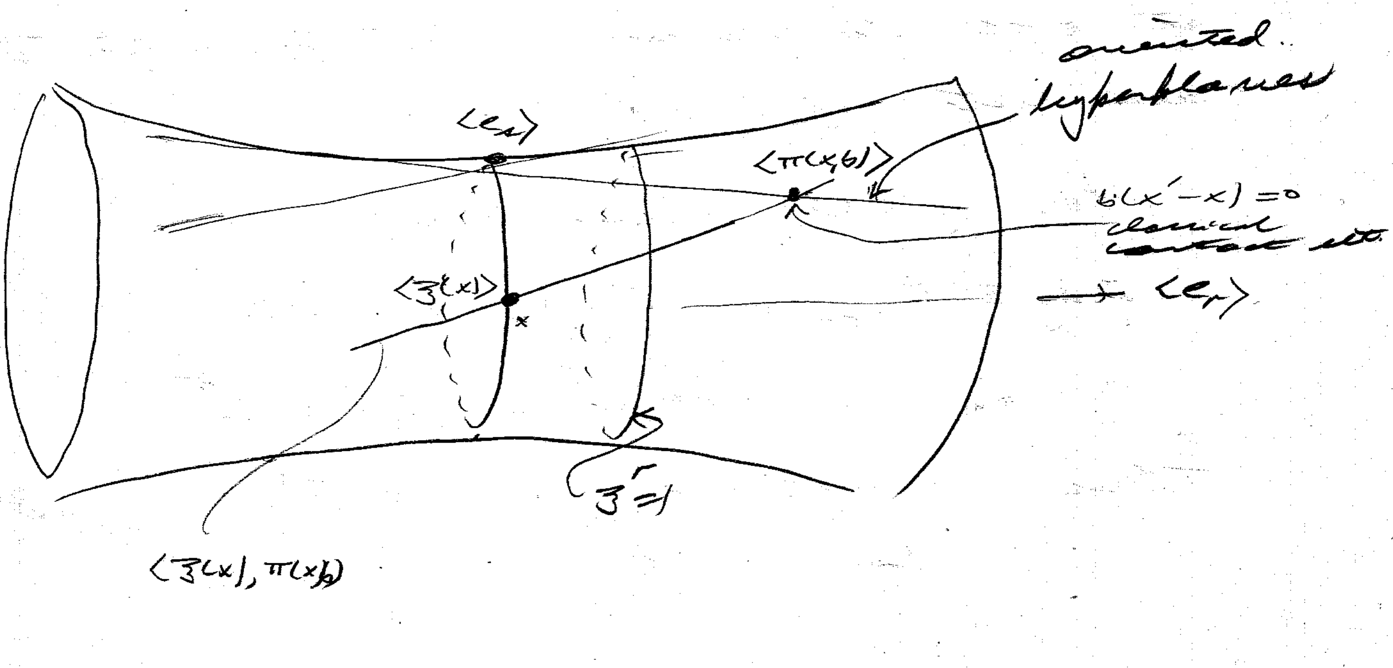
projective cotangent bundle

unoriented contact elements

$$\xi(x) = \begin{pmatrix} 1 \\ x_1 \\ \vdots \\ x_n \\ 0 \\ xx \end{pmatrix}, \quad \pi(x,b) = \begin{pmatrix} 0 \\ b_1 \\ \vdots \\ b_n \\ 1 \\ 2bx \end{pmatrix} \longleftarrow b\cdot b = 1$$

$$\begin{array}{ccccc}
B^{n-1} & \longrightarrow & E^n \times S^{n-1} & \longrightarrow & \Lambda^{2n-1} \\
\downarrow & & \downarrow \text{proj} & & \downarrow \\
\Pi^n & \cdots\longrightarrow & E^n & \text{———} & \Xi^n = \Omega^{n+1} \cap \langle e_r \rangle^{\perp}
\end{array}$$

$$\begin{array}{ccc}
(x,b) & \longrightarrow & \langle \xi(x), \pi(x,b) \rangle \\
(x,b) & & l \\
\rightsquigarrow & & \rightsquigarrow \\
x & & l \cap \langle e_r \rangle^{\perp} \\
x & \longrightarrow & \langle \xi(x) \rangle
\end{array}$$



$S^{n-1} \longrightarrow O(n;\mathbb{R})$ section such that

nth element of frame is unit vector of S^{n-1};

$\vec{b} \mapsto (\vec{a}_1(\vec{b}), \ldots, \vec{a}_{n-1}(\vec{b}), \vec{b})$.

Let

$$J(\vec{x},\vec{b}) = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ x_1 & & & & & 0 & 0 \\ \vdots & \vec{a}_1 & \cdots & \vec{a}_{n-1} & \vec{b} & \vdots & \vdots \\ x_n & & & & & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 \\ \vec{x}\cdot\vec{x} & 2\vec{a}_1\cdot\vec{x} & \cdots & 2\vec{a}_{n-1}\cdot\vec{x} & 2\vec{b}\cdot\vec{x} & 0 & 1 \end{pmatrix}$$

$J_0 \quad J_1 \longrightarrow J_n \quad J_r \quad J_s$

Then

$J : E^n \times S^{n-1} \longrightarrow O(W)$ is a section.

$L\omega = (E|dE)$ pulls back to $E^n \times S^{n-1}$ as

$$(J|dJ) = \left(J \text{ above} \;\middle|\; \begin{pmatrix} 0 & 0 & - & 0 & 0 & 0 & 0 \\ dx & da_1 & & da_{n-1} & db & & \\ 0 & 0 & & 0 & 0 & 0 & 0 \\ 0 & & & & & 0 & 0 \\ 2x\,dx & & & & 2b\,dx + x\,db & 0 & 0 \end{pmatrix} \right)$$

$xx = \vec{x}\cdot\vec{x}$ etc.

$2a_1 dx + 2x\,da_1$

$\mathfrak{Z}(\vec{x},\vec{b}) = J(\vec{x},\vec{b})\,e_0$ depends only on $\vec{x}$

$\beta(\vec{x},\vec{b}) = J(\vec{x},\vec{b})(e_n + e_r)$

$$(\beta \mid d\mathfrak{Z}) = {}^t(e_n+e_r)\,{}^tJ\,L\,dJ\,e_0$$

$$= {}^t\left(J\begin{pmatrix}0\\0\\ \vdots\\1\\1\\0\end{pmatrix}\right) L \left(dJ\begin{pmatrix}1\\0\\ \vdots\\0\\0\\0\end{pmatrix}\right) = {}^t\begin{pmatrix}0\\ \vec{b}\\ 1\\ 2\vec{b}\cdot\vec{x}\end{pmatrix} L \begin{pmatrix}0\\ d\vec{x}\\ 0\\ 2\vec{x}\cdot d\vec{x}\end{pmatrix}$$

$$= \vec{b}\cdot d\vec{x}$$

since $L = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1_n & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$

$\lambda_0 = 1$