TOPICS: Examination of Euclidean geometry, the Fifth Postulate and the history of the discovery of hyperbolic non-Euclidean geometry, hyperbolic geometry in the plane and space, angle of parallelism, Euclidean models and consistency, independence of the Parallel Postulate, philosophical implications.


PREREQUISITES: Elementary Euclidean geometry, familiarity with axiomatic mathematics

MEETS: MWF 9:00 - 9:50 USB 4030A Four units.

INSTRUCTOR: J. Fillmore

NOTE: Math 151 may be repeated for credit with a different topic.

\[
\tan \frac{T'(d)}{2} = e^{-\frac{d}{k}}
\]

"... out of nothing I have created a strange new universe."

- J. Bolyai
March 15

Tarski on geometry

Spring 1975: Non-Euclidean Geometry


0. Introduction and history
1. Foundations of Euclidean geometry
2. The Fifth Postulate
3. Historical sketch
4. Non-Euclidean plane geometry
5. Non-Euclidean solid geometry
6. Non-Euclidean plane trigonometry
7. Euclidean models and consistency
8. The unimportance of different geometries
9. Miscellaneous
10. Hilbert geometry
# 0. Introduction

Non-Euclidean (NE) geometry is a geometry in which Euclid's Fifth Postulate, the Parallel Postulate, does not hold. Its discovery solved a problem containing for 2200 years.

## Historical Time Frame

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. 300 BC</td>
<td>Euclid of Alexandria, Compilation of the Elements.</td>
</tr>
<tr>
<td>1700s</td>
<td>Saccheri (Italy) and Lambert (Germany), Precursors to NE geometry.</td>
</tr>
<tr>
<td>1824</td>
<td>Gauss (Germany, 1777-1855) Discovery of NE geometry.</td>
</tr>
<tr>
<td>1829</td>
<td>Lobachevsky (Russia, 1793-1856) of NE geometry.</td>
</tr>
<tr>
<td>1832</td>
<td>Beltrami (Italy, 1802-1860)</td>
</tr>
<tr>
<td>1854</td>
<td>Bolyai (Hungary, 1802-1860)</td>
</tr>
<tr>
<td>1854</td>
<td>Riemann (Germany, 1826-1866)</td>
</tr>
<tr>
<td>1868</td>
<td>Beltrami (Italy, 1802-1860)</td>
</tr>
<tr>
<td>1899</td>
<td>Hilbert (Germany, 1862-1943)</td>
</tr>
</tbody>
</table>

**Note:** Detailed history in Text, Chs. II, III.

## Miscellaneous

1. Discovery of NE geometry marked the beginning of modern axiomatic mathematics.
2. Prior to NE geometry, mathematics described the "real world".
3. With the advent of NE geometry, physical truth or falsity was no longer found within separated from mathematics.
1) Modern viewpoint: Postulates describe physical space - an expression of our experience. The mathematics is carefully developed and physical space necessarily follows not the mathematical propositions - at least of the postulates we now choose.

2) Kant in 1781 in "Critique of Pure Reason" declared that Euclidean geometry was inherent in the structure of the mind. The discovery of non-Euclidean geometry refuted this completely. The anticipated controversy ceased when Gauss, in his discovery of non-Euclidean geometry, discovered non-Euclidean after his death.