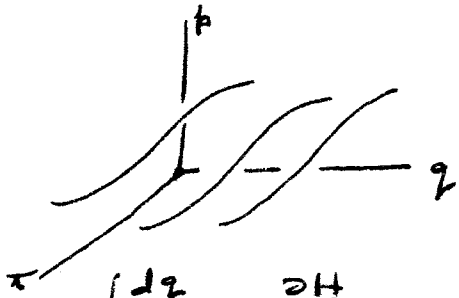
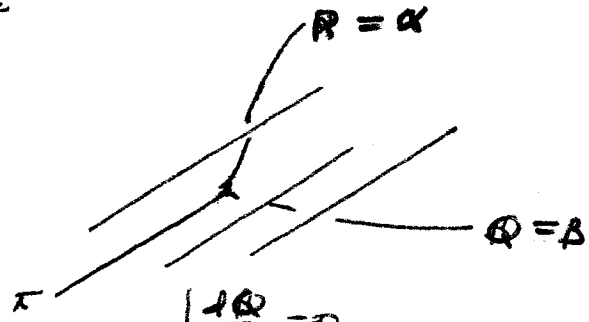
Hamilton - Jacobi equation





$$\begin{cases} \dfrac{dq}{d\tau} = \dfrac{\partial H}{\partial p} \\ \dfrac{dp}{d\tau} = -\dfrac{\partial H}{\partial q} \end{cases} \qquad \begin{cases} \dfrac{dQ}{d\tau} = 0 \\ \dfrac{dP}{d\tau} = 0 \end{cases}$$

If S satisfies $\dfrac{\partial S}{\partial \tau} + H\left(q, \dfrac{\partial S}{\partial q}, \tau\right) = 0$, then the Hamiltonian in (Q, P) coordinates is $K = 0$. i.e. Q & P are <u>constants</u> along the curve of the system. This is <u>transformation to equilibrium</u>.

Thus! Solve

$$\boxed{\frac{\partial u}{\partial \tau} + H\left(q, \frac{\partial u}{\partial q}, \tau\right) = 0}$$ <u>Hamilton - Jacobi equation</u>

for $u = S(q, \beta, \tau)$ $\qquad \beta = (\beta_1, \cdots, \beta_n)$

– a complete integral with n constants.

Then set

$$p = \frac{\partial S}{\partial q}(q, \beta, \tau), \quad \alpha = -\frac{\partial S}{\partial \beta}(q, \beta, \tau)$$

If $\det \dfrac{\partial^2 S}{\partial q \partial \beta} \neq 0$, express q & p as functions of β & α. Since β & α are <u>constants</u> of the motion, the solution curves of Hamiltons equations are!

$$\begin{cases} q = q(\beta, \alpha, \tau) \\ p = p(\beta, \alpha, \tau) \end{cases}$$ q & p as functions of β and α.