Hamilton-Jacobi equation

\[ \frac{dq}{dt} = \frac{\partial H}{\partial q} \]
\[ \frac{dp}{dt} = -\frac{\partial H}{\partial q} \]

If \( S \) satisfies \( \frac{dS}{dx} + H(q, \frac{dq}{dt}, t) = 0 \), then the Hamiltonian \( H(q, \frac{dq}{dt}, t) \) constant \( K = 0 \) is \( \mathbf{q} \times \mathbf{p} \) are constant along the curve of the system. This is stationary of equilibrium.

Thus! Since

\[ \frac{\partial q}{\partial t} + H(q, \frac{\partial q}{\partial t}, t) = 0 \]

Hamilton-Jacobi equation

For \( u = S(q, \beta, t) \), \( \beta = (\beta_1, \ldots, \beta_n) \)

a complete integral with \( n \) constants.

Thus put

\[ \mathbf{v} = \frac{\partial S}{\partial \beta} (q, \beta, t) \quad , \quad \alpha = \frac{\partial S}{\partial t} (q, \beta, t). \]

If \( \frac{dS}{d\theta} \neq 0 \), express \( q \) and \( \beta \) as functions of \( \beta \) and \( \alpha \). Since \( \beta \) and \( \alpha \) are constants of the motion, the stationary curves of Hamilton's equations are:

\[ \begin{cases} q = q(\beta, \gamma, t) & \text{as functions of } \beta \text{ and } \alpha \\ p = p(\beta, \alpha, t) & \end{cases} \]