**MATH 190A**
**WINTER 2021**

## Problem Set 10

This problem set is optional. Do not turn it in.

**1.** Let $X$ be any topological space. For each statement, determine whether it is true or false.

**(a)** Suppose $A, B \subseteq X$ are dense in $X$. Then $A \cap B$ is dense in $X$.

**(b)** Suppose $A, B \subseteq X$ are open and dense in $X$. Then $A \cap B$ is open and dense in $X$.

**2.** Let $X$ be any topological space. For each statement, determine whether it is true or false.

**(a)** Suppose $A \subseteq X$ is dense and $Y \subseteq X$ is a closed subset. Then $A \cap Y$ is dense in $Y$ with the subspace topology.

**(b)** Suppose $A \subseteq X$ is dense and $Y \subseteq X$ is an open subset. Then $A \cap Y$ is dense in $Y$ with the subspace topology.

**(c)** Suppose $A \subseteq X$ is dense and $Y \subseteq X$ is an open subset. Then $A \cap Y$ is dense in $\overline{Y}$ with the subspace topology.

**3.** Say a topological space $X$ is *locally compact* if for every point $x \in X$ there exists an open set $U \ni x$ and a closed set $V \supseteq U$ such that $V$ is compact (with the subspace topology).

Prove carefully that if $X$ is a locally compact Hausdorff space, and $A_1, A_2, \ldots$ are open and dense in $X$, then $\bigcap_{n=1}^{\infty} A_n$ is non-empty.