MATH 190A
WINTER 2021

Problem Set 7

Due by 2359 (11:59 PM) on Sunday February 21. Hand in via Gradescope.

You may discuss these problems among yourselves, but your final submitted solutions must be written by you alone, in your own words, without any other solution in front of you.

1. Let X be a topological space. Prove the following statement: if there exist open disjoint subsets $A, B \subseteq X$ such that $X = A \cup B$, $x \in A$ and $y \in B$, then x and y do not lie in the same connected component of X (i.e., $x \nsim y$ where $\sim$ is the "connected component" relation).

[Bonus question: prove that the converse does not hold.]

2. Let X, Y be topological spaces and $f\colon X \to Y$ a continuous map. For each of the following statements, give either a proof (if it is true for any X, Y and f) or a counterexample (if it is false for some X and Y and f).

(a) If $Z \subseteq X$ is a connected component of X then $f(Z)$ is a connected component of Y.

(b) If $Z \subseteq X$ is a connected component of X then there exists a connected component $W \subseteq Y$ of Y such that $f(Z) \subseteq W$.

(c) If $W \subseteq Y$ is a connected component of Y then $f^{-1}(W)$ is a connected component of X.

(d) If $W \subseteq Y$ is a connected component of Y and Z is a connected component of X then either $Z \subseteq f^{-1}(W)$ or $Z \cap f^{-1}(W) = \emptyset$ (i.e., $f^{-1}(W)$ is a union of connected components of X).