Math 200b (Winter 2016) - Homework 2

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The first four exercises can be found in Dummit and Foote: p. 545, #1 and p. 567, ## 3, 4, 9.

Exercise 1. Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 - 2$.

Exercise 2. Determine the fixed field of complex conjugation on $\mathbb{C}$.

Exercise 3. Prove that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic.

Exercise 4. Determine the fixed field of the automorphism $t \mapsto t + 1$ of $k(t)$.

Exercise 5. Find the following minimal polynomials: (a) $2 - 3i$ over $\mathbb{R}$, (b) $\sqrt{2} + \sqrt{3}$ over $\mathbb{Q}$ and (c) $1 + \sqrt{2}$ over $\mathbb{Q}(\sqrt{2} + \sqrt{3})$.

Exercise 6. Find all automorphisms of the following fields: (a) $\mathbb{Q}(\sqrt{2} + \sqrt{3})$ and (b) $\mathbb{Q}(\sqrt[3]{2})$.