Math 200b (Winter 2016) - Homework 4

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Exercise 1. Determine the Galois groups of the following polynomials over $\mathbb{Q}$:
(a) $t^2 - t + 1$, (b) $t^2 - 4$, (c) $t^2 + t + 1$, (d) $t^2 - 27$.

Exercise 2. Determine the Galois groups of the following polynomials over the indicated fields: (a) $t^3 - 10$ over $\mathbb{Q}(\sqrt{2})$, (b) $t^2 - 5$ over $\mathbb{Q}(\sqrt{-5})$.

Exercise 3. Let f be an irreducible polynomial of degree 3 over some field F. Prove that the splitting field K of f contains at most one subfield of degree 2 over F.

Exercise 4. (a) Prove that $t^9 - 1$ and $t^7 - 1$ have isomorphic Galois groups over $\mathbb{Q}$.
(b) Prove that the Galois groups of $t^{10} - 1$ and $t^8 - 1$ over $\mathbb{Q}$ are not isomorphic.

Exercise 5. Let E be the splitting field of $t^6 - 1$ over $\mathbb{Q}$. Show that there is no field K with the property $\mathbb{Q} \subsetneq K \subsetneq E$.

Exercise 6. Let p be a prime number and k any strictly positive integer. Prove that $\Phi_{p^k}(t) = \Phi_p(t^{p^{k-1}})$, where Φ_n is the nth cyclotomic polynomial. Use this to find $\Phi_8(t)$ and $\Phi_{27}(t)$.