# Math 200b (Winter 2016) - Homework 5 

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Exercise 1. Let $K$ be a finite extension of $\mathbb{Q}$. Prove that there are only finitely many roots of 1 in $K$.

Exercise 2. Is the equation $x^{5}+x-1=0$ over $\mathbb{Q}$ solvable by radicals?
Exercise 3. Prove that $\mathbb{Q}(\sqrt[3]{2})$ is not a subfield of any cyclotomic extension of $\mathbb{Q}$.
Exercise 4. Let $G$ be a finite abelian group. Prove that there exists a Galois extension of $\mathbb{Q}$ whose Galois group is $G$.
Hint: Show that $G$ is a quotient of $(\mathbb{Z} / m \mathbb{Z})^{n}$ for some appropriate integers $m, n$.
Exercise 5. For each $n \geq 1$, find a field extension $K / F$ such that $\operatorname{Gal}(K / F) \cong S_{n}$.

