Math 200b (Winter 2016) - Homework 5

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Exercise 1. Let K be a finite extension of \mathbb{Q} . Prove that there are only finitely many roots of 1 in K.

Exercise 2. Is the equation $x^5 + x - 1 = 0$ over \mathbb{Q} solvable by radicals?

Exercise 3. Prove that $\mathbb{Q}(\sqrt[3]{2})$ is not a subfield of any cyclotomic extension of \mathbb{Q} .

Exercise 4. Let G be a finite abelian group. Prove that there exists a Galois extension of \mathbb{Q} whose Galois group is G.

Hint: Show that G is a quotient of $(\mathbb{Z}/m\mathbb{Z})^n$ for some appropriate integers m, n.

Exercise 5. For each $n \ge 1$, find a field extension K/F such that $\operatorname{Gal}(K/F) \cong S_n$.