Math 200b (Winter 2016) - Homework 7

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Exercise 1. Let D be a division algebra. Prove that the set D^n of n-tuples of elements of D is an irreducible right $\mathrm{M}_n(D)$-module.

Exercise 2. Let A be an algebra over some field F. Prove that A is simple if and only if the matrix algebra $\mathrm{M}_n(A)$ is simple.

Exercise 3. Let A be a simple algebra (with identity). Prove that the center $Z = \{z \in A \mid za = az \text{ for all } a \in A\}$ of A is a field.

Exercise 4. Let D_1, D_2 be division algebras. Prove that $\mathrm{M}_{n_1}(D_1) \cong \mathrm{M}_{n_2}(D_2)$ if and only if $n_1 = n_2$ and $D_1 \cong D_2$.

Exercise 5. Let A be an algebra over some field F. Let V be a faithful irreducible right A-module and let $D = C(V)$ be the centralizer of V. Prove that either $A \cong \mathrm{M}_n(D)$ for some n, or for every $k \geq 1$, A contains a subalgebra having $\mathrm{M}_k(D)$ as a homomorphic image.