SECOND HOMEWORK, DUE JANUARY 25TH

1. (i) Show that if R is Noetherian then so is $R[\![x]\!]$.
(ii) Prove that if R is Noetherian then so is $R[\![x_1, x_2, \dots, x_n]\!]$, where the last term is defined appropriately.
2. Let M be a Noetherian R-module. If $\phi\colon M \longrightarrow M$ is a surjective R-linear map, prove that ϕ is an automorphism. (*Hint, consider the submodules,* $\mathrm{Ker}(\phi^n)$).
3. Let k be a field and let $S \subset k[x_1, x_2, \dots, x_n]$ be a set of polynomials in the variables $x_1, x_2, \dots, x_n$ with coefficients in k. The **zero set** of S, $V(S) \subset k^n$ is just the set of all $(a_1, a_2, \dots, a_n) \in k^n$ such that $f(a_1, a_2, \dots, a_n) = 0$ for all $f(x_1, x_2, \dots, x_n) \in S$.
Show that we may find a finite subset $S_0 \subset S$ such that $V(S) = V(S_0)$.
(*Hint: Observe that $V(S) = V(I)$ where I is the ideal generated by S.*)