FIFTH HOMEWORK, DUE FEBRUARY 13TH

1. Let M and N be two modules over a PID R. Show that the rank of $M \oplus N$ is equal to the sum of the ranks of M and N.

2. How many rational canonical forms are there for 5×5 matrices over the field with two elements?

3. What are the eigenvalues of the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}?$$

4. Find the rational canonical form of

$$\begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

5. Find the Jordan canonical form of

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}.$$