SIXTH HOMEWORK, DUE FEBRUARY 22ND

1. What is $[\bar{\mathbb{Q}} : \mathbb{Q}]$?
Let L/K be a field extension. Let G be the set of automorphisms of L that fix K.
2. Show that G is naturally a group. G is called the **Galois group** of L/K and is denoted $\mathrm{Gal}(L/K)$.
3. Let M be an intermediary subfield. Let H be the set of all automorphisms of L over K that fix M. Show that H is a subgroup of G.
4. Let H be a subgroup of G. Let M be the subset of all elements of L fixed by H. Then M is an intermediary field, denoted L^H, called the fixed field of H.
5. Show that both assignments are inclusion reversing, that is, if $H \subset K$ are two subgroups, then $L^K \subset L^H$ and if $M \subset N$ are two intermediary fields, then $\mathrm{Gal}(L/N) \subset \mathrm{Gal}(L/M)$.
6. Given a subgroup H of G, let $K = \mathrm{Gal}(L/L^H)$. Show that $H \subset K$.
7. Given an intermediary field M, let $N = L^{\mathrm{Gal}(L/M)}$. Show that $M \subset N$.
8. Let $L = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$. Find all the intermediary fields and subgoups of the Galois group. Compare.
9. Construct subfields of $\mathbb{C}/\mathbb{Q}$ which are splitting fields for $t^3 - 1$, $t^4 + 5t^2 + 6$, $t^6 - 8$ and find the degrees of the extensions.
10. Let $f(x)$ be a polynomial of degree n over a field K, with splitting field L/K. Show that the degree of the extension L/K divides $n!$.