SEVENTH HOMEWORK, DUE WEDNESDAY MARCH 1ST

1. Show that we can extend the definition of the formal derivative to $K(t)$ be defining

$$D(f/g) = \frac{(Df \cdot g - f \cdot Dg)}{g^2}.$$

Verify the relevant properties of D.
2. Which of the polynomials x^3+1, x^2+x-1, $x^6+x^5+x^4+x^3+x^2+x+1$ and $7x^5+x-1$ are separable, considered over the fields, $\mathbb{Q}$, $\mathbb{C}$, $\mathbb{F}_2$, $\mathbb{F}_3$, $\mathbb{F}_5$, $\mathbb{F}_7$ and $\mathbb{F}_{17}$?
3. Which of the extensions

(1) $\mathbb{Q}(t)/\mathbb{Q}$,
(2) $\mathbb{Q}(\sqrt{-5})/\mathbb{Q}$,
(3) $\mathbb{Q}(\alpha)/\mathbb{Q}$, where α is the real seventh root of 5,
(4) $\mathbb{Q}(\alpha, \sqrt{5})/\mathbb{Q}$, where α is the real seventh root of 5,

are normal?
4. Show that every extension of degree two is normal.
5. Show that if L/K is separable and M is an intermediary field, then both L/M and M/K are separable extensions.
6. Is every normal extension of a normal extension, normal?
7. Find a finite extension that is not primitive.
8. Suppose that $L = K(\alpha)/K$ is a primitive extension, where α is transcendental over K. Show that L is not algebraically closed.
9. Suppose that L/K is algebraic. Show that there is a greatest intermediary field M, for which M/K is normal.
10. Suppose that L/K is a field extension and that M_1/K and M_2/K are two normal intermediary field extenstions. Show that both $K(M_1, M_2)$ and $M_1 \cap M_2$ are normal.
Challenge Problem: 11. How many irreducible polynomials of degree d are there over a field with $q = p^k$ elements?
12. Let $\gamma = \sqrt{2+\sqrt{2}}$. Show that $\mathbb{Q}(\gamma)/\mathbb{Q}$ is normal, with cyclic Galois group. Show that $\mathbb{Q}(\gamma, i) = \mathbb{Q}(\phi)$, where $\phi^4 = i = \sqrt{-1}$.