

HOMEWORK 5

DUE 12 MAY 2017

1. Read about the tensor product of algebras in Atiyah-Macdonald pages 30–31. (Note that there is a typo in the definition of the ring homomorphism $A \rightarrow D$ on page 31.) Prove that $R_1 \otimes_{\mathbb{Z}} R_2$ is the coproduct of R_1 and R_2 in the category of commutative rings.
2. Let d be a square-free integer. We denote by \mathcal{O}_d the integral closure of \mathbb{Z} in the quadratic field $K = \mathbb{Q}(\sqrt{d})$. Show that

$$\mathcal{O}_d = \begin{cases} \mathbb{Z}[\sqrt{d}] & \text{if } d \equiv 2, 3 \pmod{4} \\ \mathbb{Z}\left[\frac{1+\sqrt{d}}{2}\right] & \text{if } d \equiv 1 \pmod{4}. \end{cases}$$

3. Atiyah-MacDonald, Chapter 3, Exercise 13.
4. Atiyah-MacDonald, Chapter 3, Exercise 23.
5. Atiyah-MacDonald, Chapter 5, Exercise 1.
6. Atiyah-MacDonald, Chapter 5, Exercise 2.
7. Atiyah-MacDonald, Chapter 5, Exercise 3.
8. Atiyah-MacDonald, Chapter 5, Exercise 4.