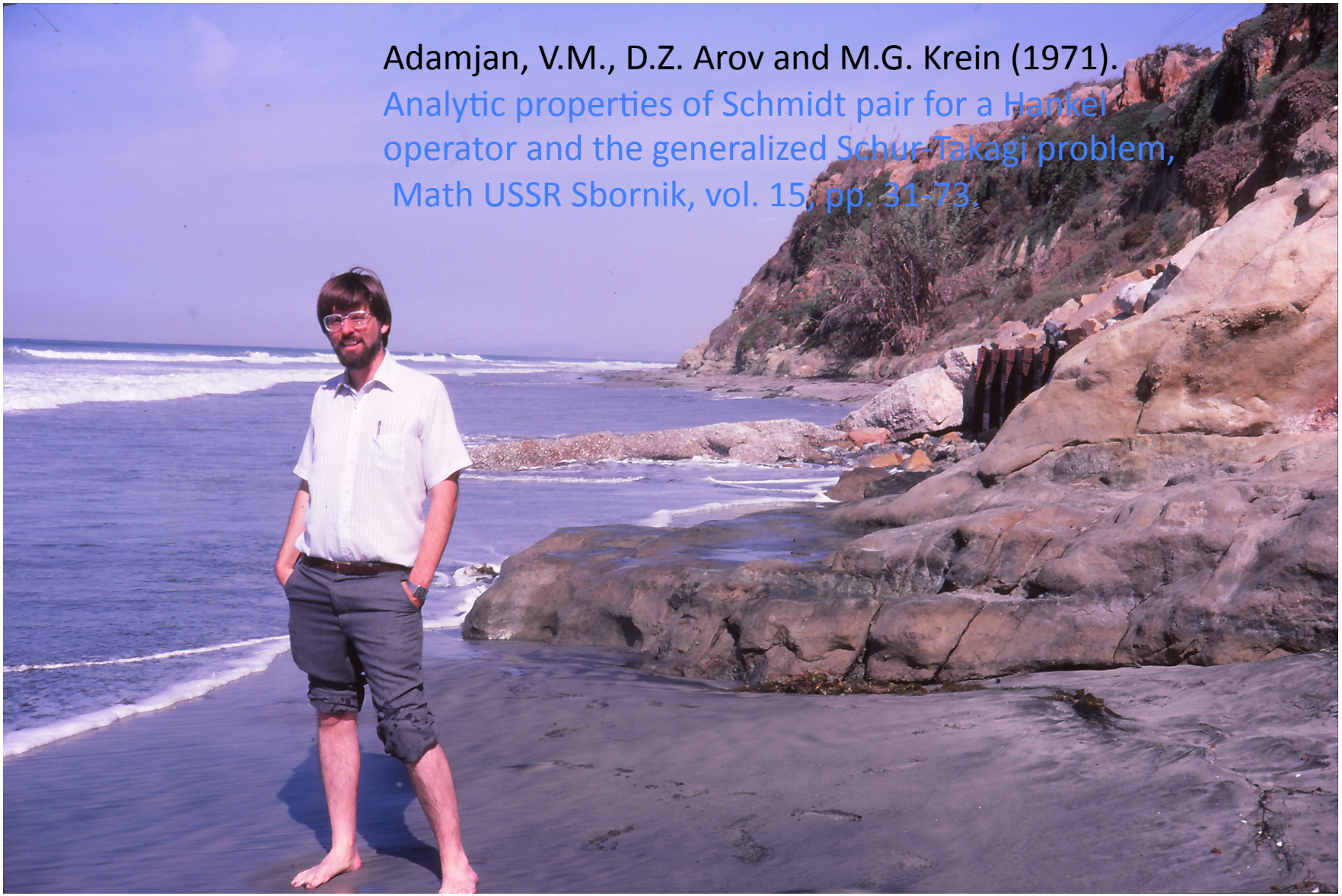
UNIVERSITY OF CAMBRIDGE

Some Computational Questions in Robust Stabilization and Fault Detection

Keith Glover (with thanks to Xiaochuan Yuan, Ann Dowling, Andreas Varga)

October 3rd, 2010, UCSD HeltonFest

Cambridge University Engineering Department

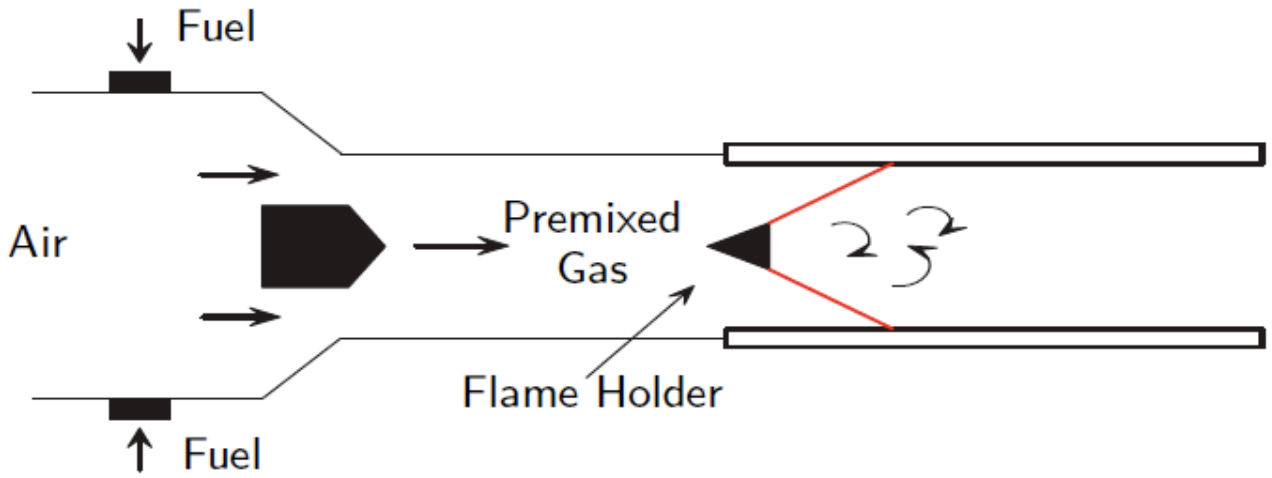


Mathematics as practiced at UCSD 1986

Outline

- Thermoacoustic Oscillation Control
  - Phenomenon Mechanism
  - Current Status
  - Methods and Results
  - Conclusion and Future work
- Fault Detection

Generic Problem: Flame burning in a fluid



The interaction between unsteady heat release $Q(t)$ and pressure/velocity fluctuations $p(t)/u(t)$ causes self-excited oscillations

Current Status

1. Active control proved one of successful applications of control technology in fluid systems(Annaswamy,2002[2], Dowling,2005[7])
2. Lean prevaporised premixed technology is a big challenge(Dowling,2002[6])
3. Modeling problems with imperfectly premixed combustion(fully premixed combustion).

Current Status

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Research purpose

Modeling investigation of imperfectly premixed combustion for the purpose of active control(Papachristodoulou & Dowling,2007[9])

Flame dynamics

Development

1. Empirical(Bloxsidge,1988)[4]

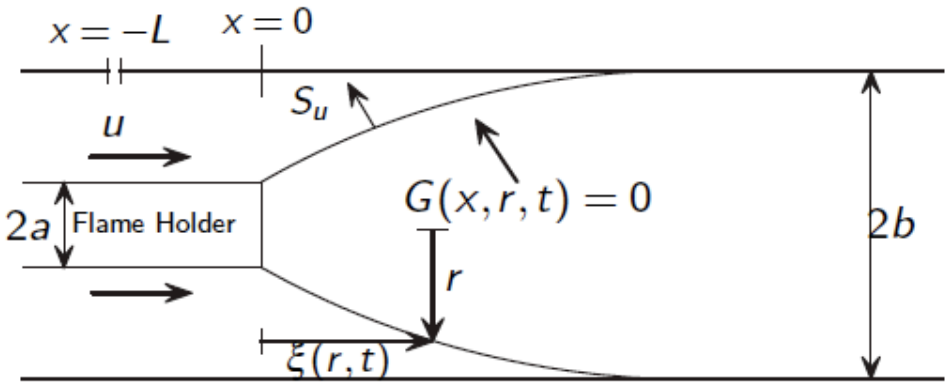
$$\frac{\hat{Q}(\omega)}{\bar{Q}} = \frac{1}{1+j\omega\tau}\frac{\hat{u}(\omega)}{\bar{u}}$$

More accurately

$$\frac{\hat{Q}(\omega)}{\bar{Q}} = \frac{1}{(1+j\omega\tau_1)(1+j\omega\tau)}\frac{\hat{u}(\omega)}{\bar{u}}, \quad (\tau_1 \ll \tau)$$

Flame dynamics

2 G-equation Flame model(Fleifil et al. 1996[8]; Dowling 1999[5])



Flame surface $G(x,r,t) = x - \xi(r,t) = 0$ moves with fixed velocity S_u into unburnt gas according to the so-called G equation

$$\frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = S_u |\nabla G|$$

$$\frac{\partial \xi}{\partial t} = u - v\frac{\partial \xi}{\partial r} - S_u\sqrt{1 + (\frac{\partial \xi}{\partial r})^2}$$

Flame dynamics

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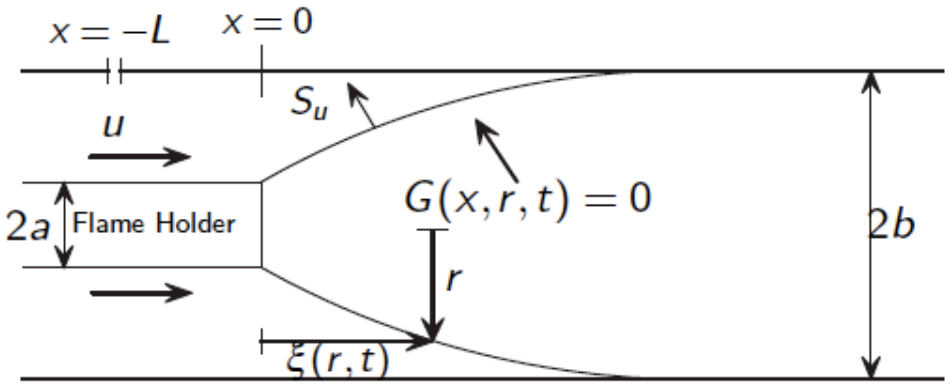

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Assumption:
$S_u = \text{Const}$ and $Q(t) \propto A(t)$
Simple Geometry, Fully Premixed

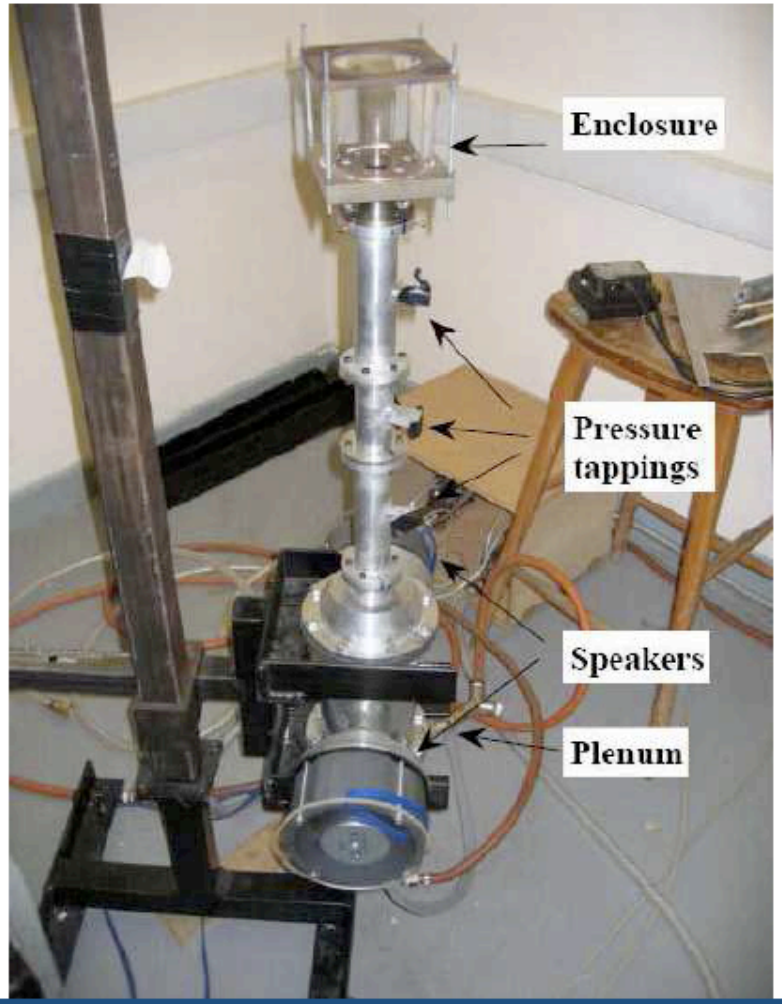
Acoustics

Principles

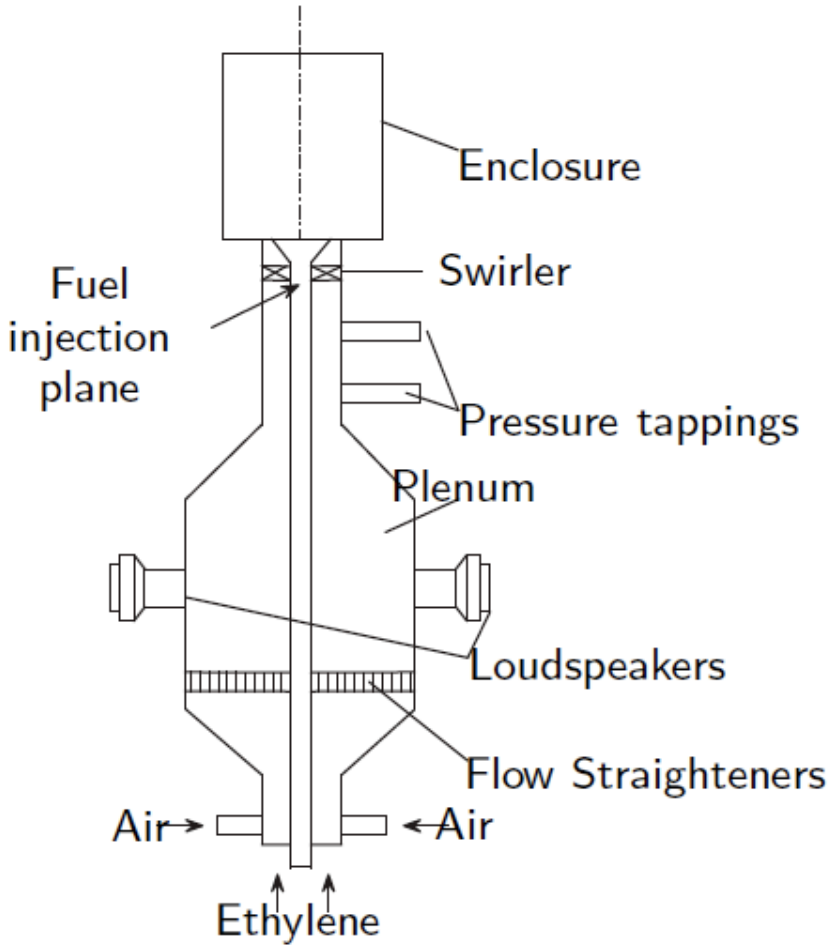
Conservation of mass, momentum and energy

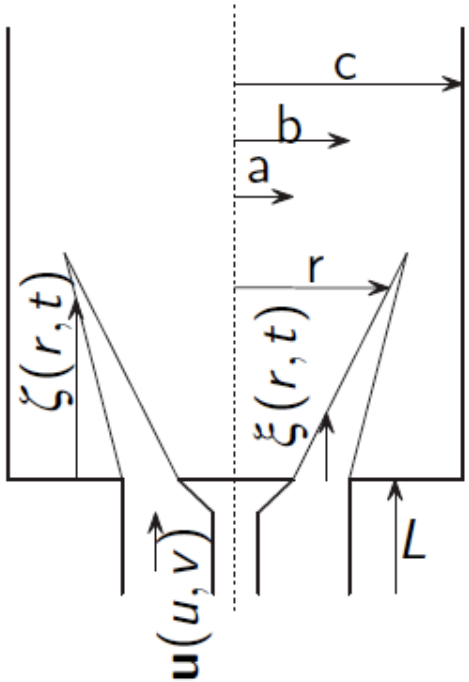
$$\frac{1}{\bar{c}}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = \frac{\gamma - 1}{\bar{c}^2}\frac{\partial Q'}{\partial t}$$

Experiments Apparatus(Balachandran,2005)[3]



Experiment(Balachandran:2005)[3]





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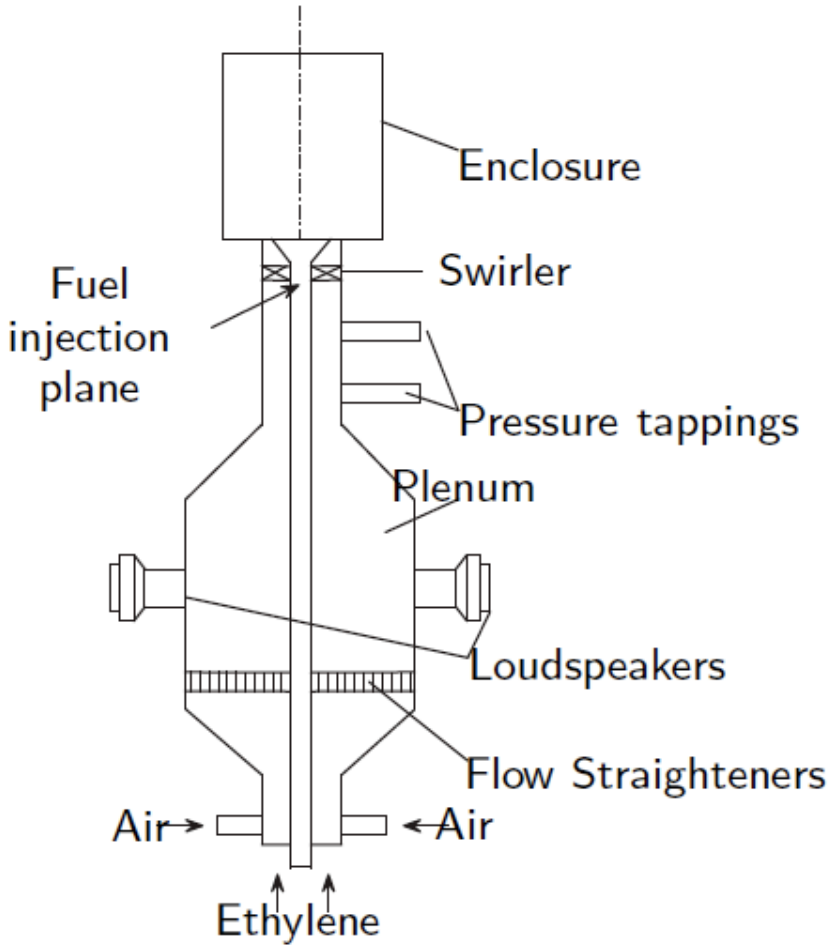
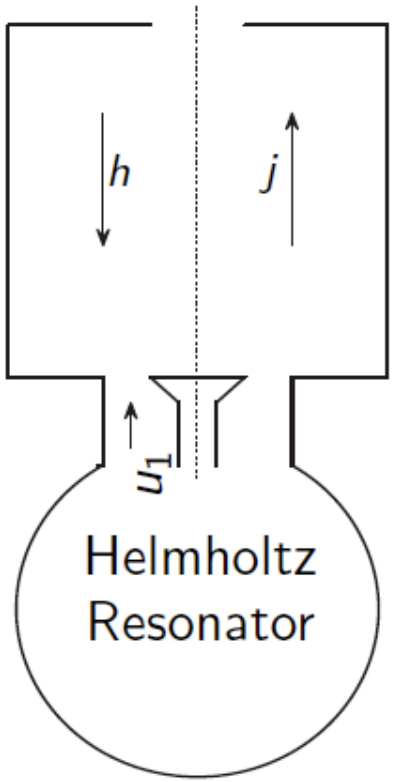
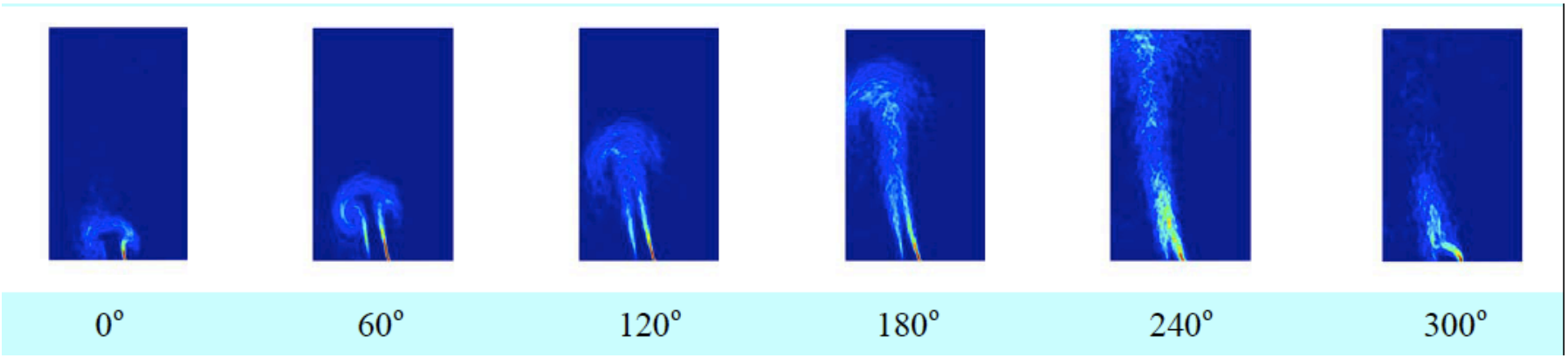




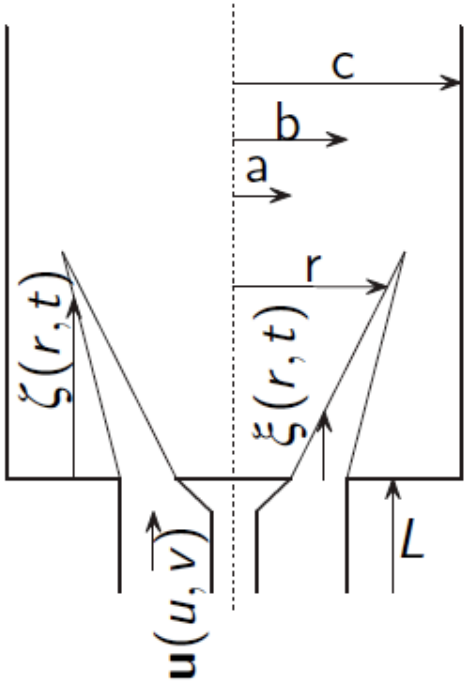
Experimental Results (Balachandran,2005)[3]



Forced response at 160 Hz excitation

Flame dynamics

1. Imperfectly premixed flame



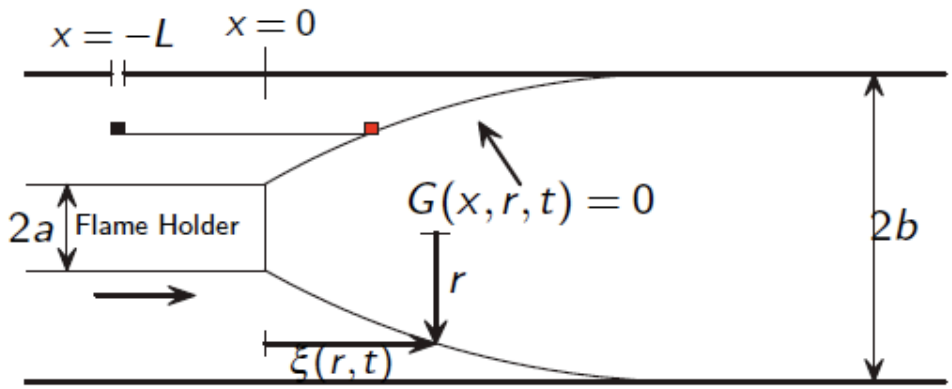
$$\begin{cases} \dfrac{\partial \xi}{\partial t} = u - v\dfrac{\partial \xi}{\partial r} - S_u\sqrt{1+\left(\dfrac{\partial \xi}{\partial r}\right)^2} \\ \dfrac{\partial \zeta}{\partial t} = u - v\dfrac{\partial \zeta}{\partial r} + S_u\sqrt{1+\left(\dfrac{\partial \zeta}{\partial r}\right)^2} \end{cases}$$

Equivalence/Fuel-air ratio ϕ

$$S_u(\phi) = k_1\phi^{k_2}e^{-k_3(\phi-k_4)^2}$$

(G. Abu-orf,1996)[1]

2. Constant and Time-varying time-delays



$$\begin{cases} \phi(r,t) = \phi_0(t - \bar{\tau}(r)) \\ \phi_0 = \dfrac{|\bar{u}|\bar{\phi}}{|u|} \end{cases}$$

where

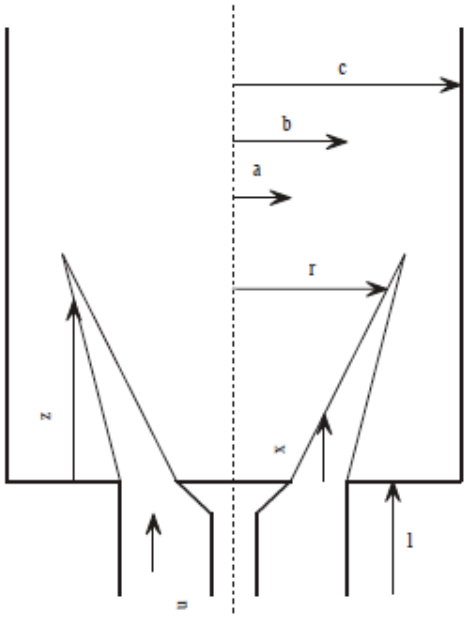
$$\bar{\tau}(r) = \frac{L + \bar{\xi}}{|\bar{u}|}$$

$$L + \xi(r,t) = \int_{t-\tau(r,t)}^{t} u(t')dt'$$

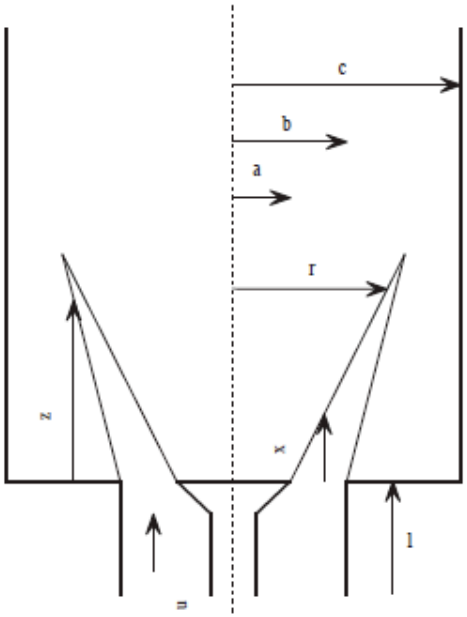
$$\Rightarrow \frac{\partial \tau}{\partial t}(r,t) = 1 - \frac{u(t) - \partial\xi/\partial t}{u(t - \tau(r,t))}$$

Flame dynamics

$$u = A\sin(2\pi f_i t)$$



$A = 0.1, f_i = 300$ Hz



$A = 0.3, f_i = 300$ Hz

Model linearization for controller synthesis

Derive a linear flame model $G_f(s)$, which is the transfer function from the incoming velocity perturbation $\hat{u}$ to global heat release variation $\hat{Q}$

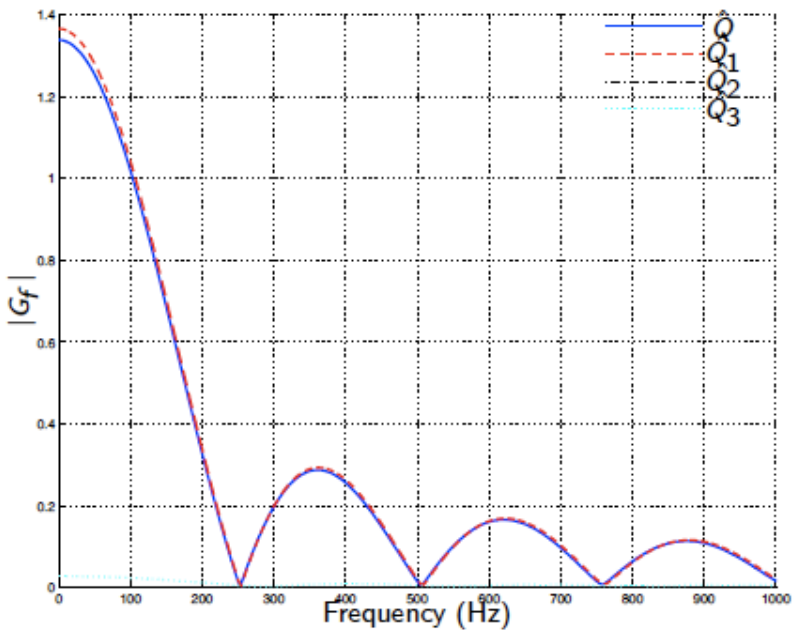


Fig. Fully Premixed Flames

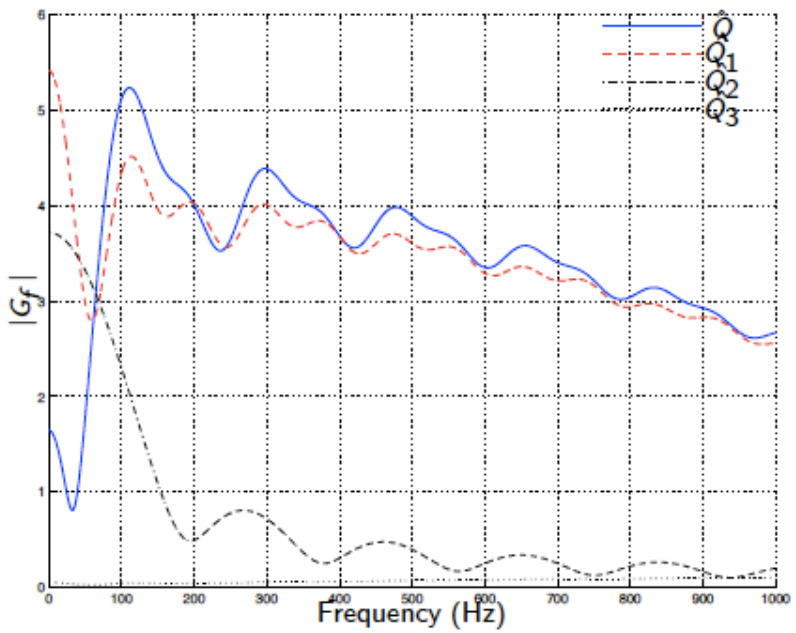
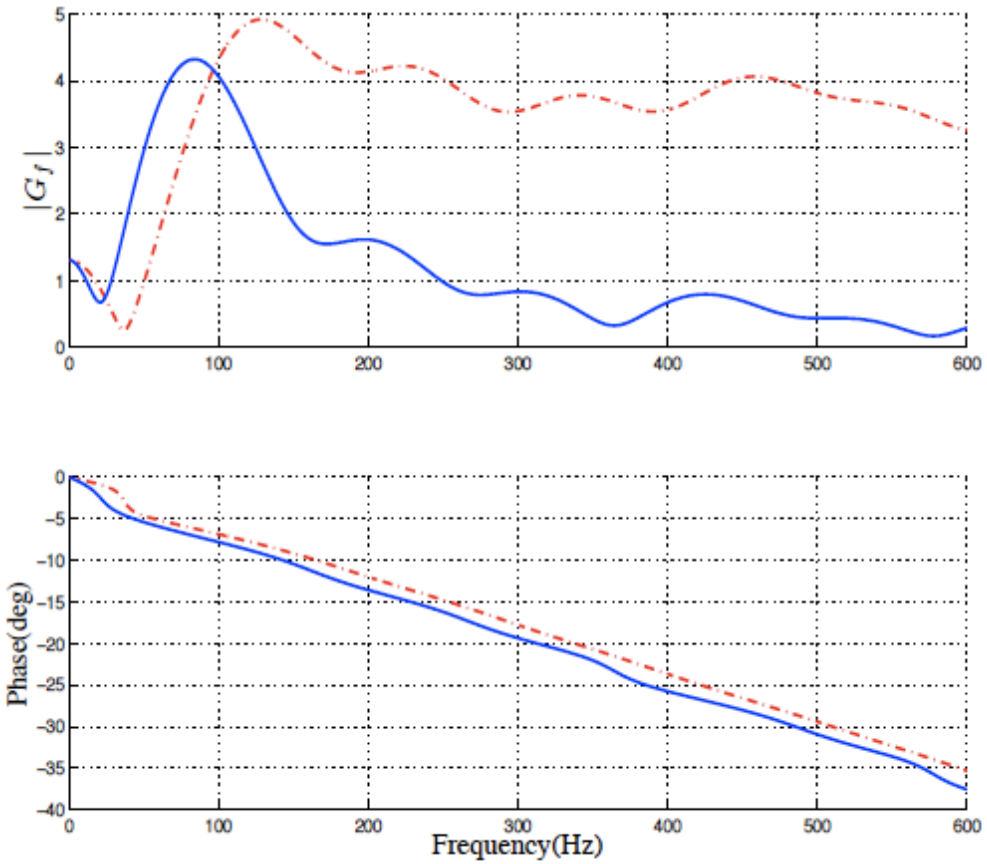


Fig. Imperfectly Premixed Flames

However the high frequency gain is seen to be too high relative to the experimental results.
Introducing some mixing gives better match.



Acoustic Model

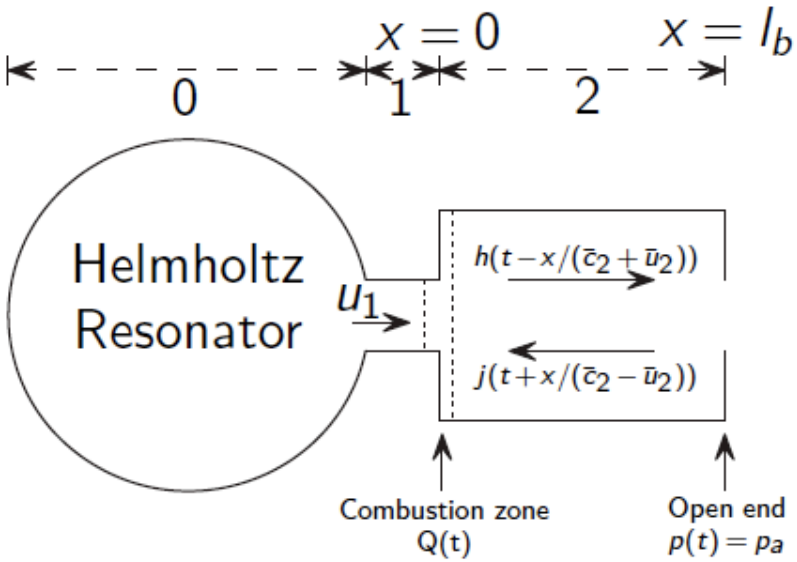


Fig. Simplified diagram of the duct geometry and flow

1. Helmholtz resonator

$$\hat{u}_1 = G_h(s)\hat{p}_1$$

2. Connect HR with downstream tube(Wave-based method)

$$\frac{\hat{u}_1}{\bar{u}_1} = G_{ac}(s)\frac{\hat{Q}}{\bar{Q}}$$

Acoustic Model

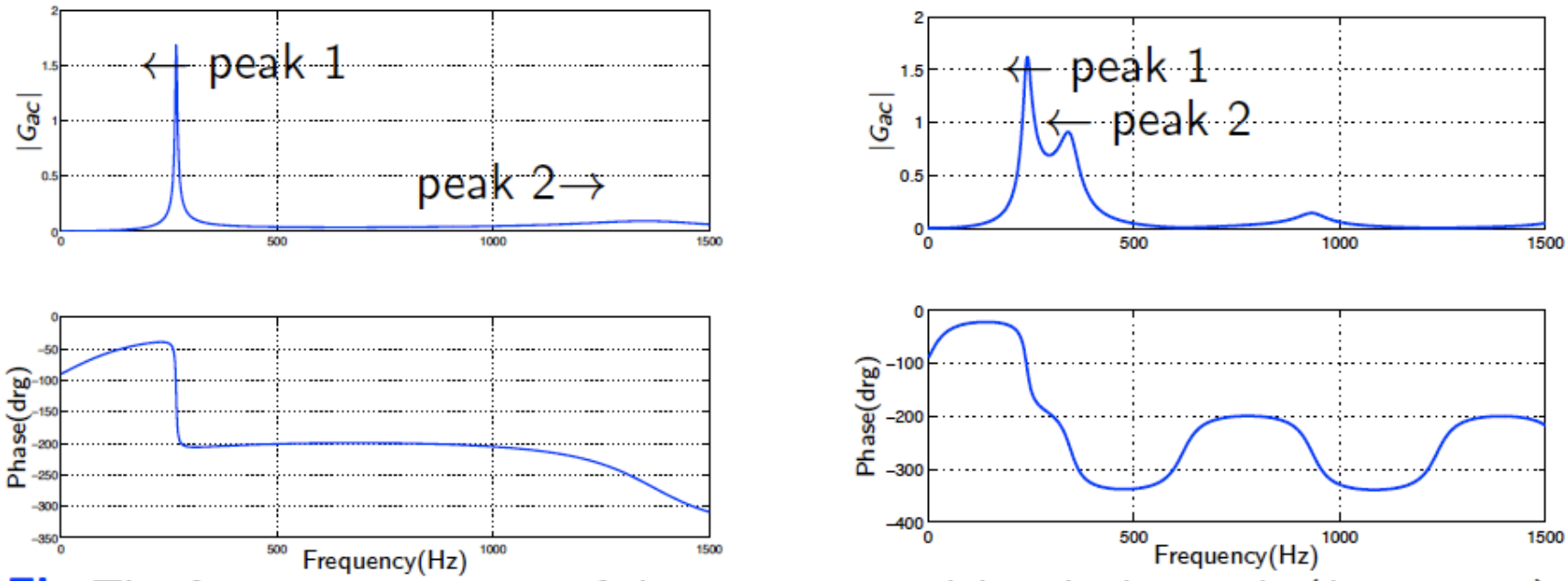


Fig. The frequency response of the acoustic model with short tube($l_b = 80mm$) and long tube($l_b = 350mm$)

Coupling of the flame and acoustic model

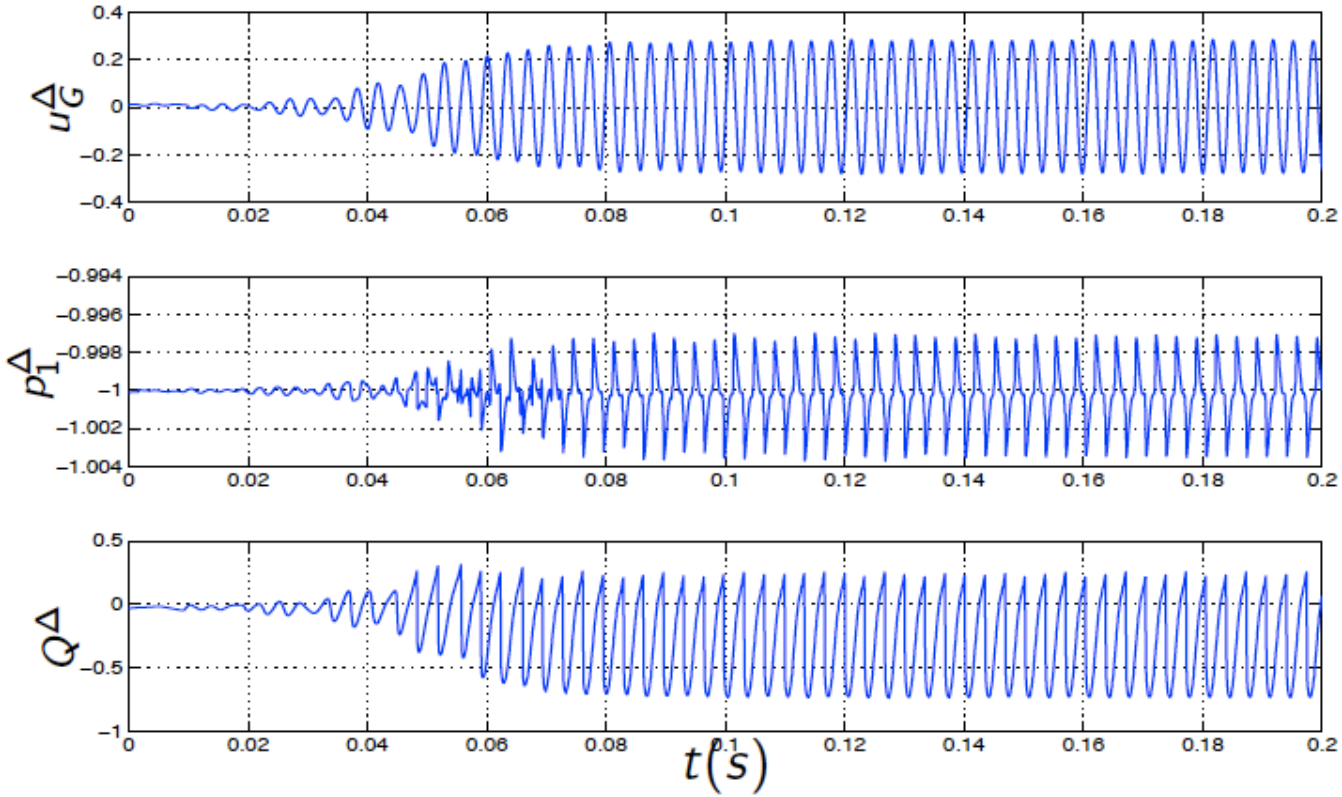


Fig. Simulations of combustion instabilities: The fractional perturbations in $u_G(t)$(the fluid velocity), $p_1(t)$(the pressure), and $Q(t)$(heat release)

Stabilization of Nominal Model

Controller designed for linearized model with discretized pde using $\mathscr{H}_\infty$ techniques.

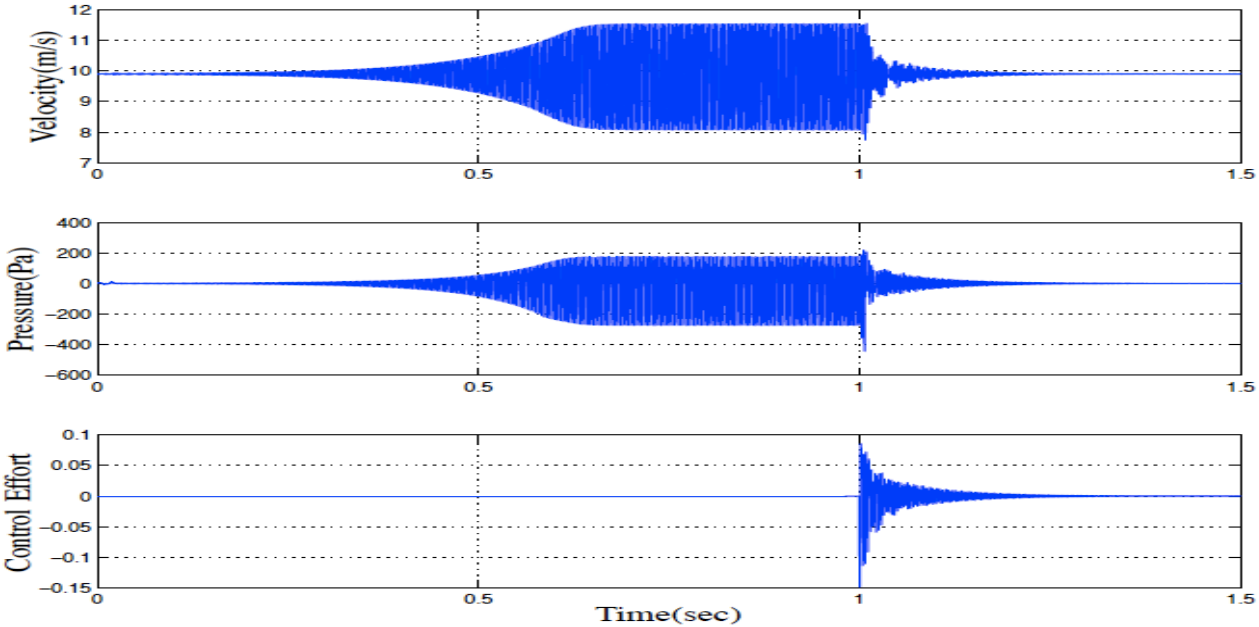


Fig. Simulation of stabilization of combustion instabilities: The fractional perturbations in $u_G(t)$(the fluid velocity), $p_1(t)$(the pressure), and control effort

Analysis Tools - Integral Quadratic Constraints (Megretski and Rantzer)

- Approximate bounds on nonlinear deviations from linearized but discretized model.
- With N discrete points for r then we have a Δ block of dimension $12N \times 6N$.
- Performance in both speed and robustness of the current LMI solvers were challenged by this scale of problem.
- Similar tools for delay approximation.
- Convergence as N becomes large?

A Fault Detection Problem (Andras Varga, DLR)

$$y = G_u u + G_d d + G_w w + G_f f$$

where

- y - ouptut (measured)
- u - input (measured)
- d - disturbance (not measured)
- w - noise (not measured)
- f - fault (not measured)

Approx. Fault Detection Problem (AFDP): Find R

$$r \quad = \quad R \begin{bmatrix} y \\ u \end{bmatrix} = R_u u + R_d d + R_w w + R_f f$$

such that $R_u = 0$ and $R_d = 0$ and (i) $\|R_w\|_{2/\infty} \leq \gamma$, (ii) $\|R_{fj}\|_{2/\infty} \geq \beta \quad \forall j$.

Standard manipulations reduce the problem to finding $Q \in \mathscr{H}_\infty$ such that

(i) $\|QM_{wo}\|_{2/\infty} \leq \gamma$

(ii) $\|QN_{fj}\|_{2/\infty} \geq \beta \quad \forall j.$

where M_{wo} is an outer function.
Want the maximize β/γ the sensitivity to fault versus noise.

If $\|.\|_\infty$ in (i) then

- if $M_{wo}^{-1} \in \mathscr{H}_\infty$ then optimal solution is $Q = \gamma M_{wo}^{-1}$.
- otherwise can typically make β arbitrarily large as "$Q \to M_{wo}^{-1}$".

Optimal solution with fixed form $Q = D + C(sI - A)^{-1}B$ with (A, B) given but (C, D) to be chosen?
Let $X = [C \quad D]' \; [C \quad D]$ then

- $\|QM_{wo}\|_\infty \leq \gamma$ is equivalent to an LMI in X.
- $\|QN_{fj}\|_2 \geq \beta$ is equivalent to an LMI in X.
- but $\|QN_{fj}\|_\infty \geq \beta$ is NOT equivalent to an LMI.

[Note that although there appears to be a rank condition on X a subsequent spectral factorization can remove this]

Question: is the problem with $\|QM_{wo}\|_\infty / \|QN_{fj}\|_\infty$ over $[C\ D]$ inherently non-convex?

Example: Q a 2×2 constant. Let $X = Q'Q$.

(i) $X \leq \begin{bmatrix} \frac{1+\alpha\omega^2}{1+\omega^2} & 0 \\ 0 & \frac{\alpha+\omega^2}{1+\omega^2} \end{bmatrix} \quad \forall \omega.$

(ii) $\begin{bmatrix} \frac{1+j\omega}{1-j\omega} & 1 \end{bmatrix} X \begin{bmatrix} \frac{1-j\omega}{1+j\omega} \\ 1 \end{bmatrix} \geq \beta$ for some ω.

(iii) $\begin{bmatrix} 1 & -\frac{1+j\omega}{1-j\omega} \end{bmatrix} X \begin{bmatrix} 1 \\ -\frac{1-j\omega}{1+j\omega} \end{bmatrix} \geq \beta$ for some ω.

Some manipulation gives $\beta_{\max} = \frac{4\alpha}{1+\alpha}$ and $X_{\text{opt}} = \frac{\alpha}{1+\alpha} \begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}$.

Two disjoint sets of feasible solutions for $\beta < \beta_{\max}$.

Mathematics and Engineering as practiced at UCSD
2010 and beyond

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