Some Computational Questions in Robust Stabilization and Fault Detection

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Outline

- Thermoacoustic Oscillation Control
  - Phenomenon Mechanism
  - Current Status
  - Methods and Results
  - Conclusion and Future work
- Fault Detection
Generic Problem: Flame burning in a fluid

The interaction between unsteady heat release $Q(t)$ and pressure/velocity fluctuations $p(t)/u(t)$ causes self-excited oscillations.
Current Status

1. Active control proved one of successful applications of control technology in fluid systems (Annaswamy, 2002[2], Dowling, 2005[7])

2. Lean prevaporised premixed technology is a big challenge (Dowling, 2002[6])

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Research purpose

Modeling investigation of imperfectly premixed combustion for the purpose of active control (Papachristodoulou & Dowling, 2007[9])
Flame dynamics

Development

1. Empirical (Bloxsidge, 1988) \[4\]

\[
\frac{\hat{Q}(\omega)}{\bar{Q}} = \frac{1}{1 + j\omega \tau} \frac{\hat{u}(\omega)}{\bar{u}}
\]

More accurately

\[
\frac{\hat{Q}(\omega)}{\bar{Q}} = \frac{1}{(1 + j\omega \tau_1)(1 + j\omega \tau)} \frac{\hat{u}(\omega)}{\bar{u}}, \quad (\tau_1 \ll \tau)
\]
Flame dynamics

2 G-equation Flame model (Fleifil et al. 1996[8]; Dowling 1999[5])

\[ x = -L \quad x = 0 \]

\[ u \quad S_u \]

\[ \frac{\partial G}{\partial t} + u \cdot \nabla G = S_u |\nabla G| \]

\[ \frac{\partial \xi}{\partial t} = u - v \frac{\partial \xi}{\partial r} - S_u \sqrt{1 + \left(\frac{\partial \xi}{\partial r}\right)^2} \]

Flame surface \( G(x, r, t) = x - \xi(r, t) = 0 \) moves with fixed velocity \( S_u \) into unburnt gas according to the so-called G equation.
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Assumption:
- $S_u = \text{Const}$ and $Q(t) \propto A(t)$
- Simple Geometry, Fully Premixed
**Principles**

Conservation of mass, momentum and energy

\[
\frac{1}{c} \frac{\partial^2 \rho'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = \frac{\gamma - 1}{c^2} \frac{\partial Q'}{\partial t}
\]
Experiments Apparatus (Balachandran, 2005)[3]
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Experimental Results (Balachandran, 2005)[3]

Forced response at 160 Hz excitation
Flame dynamics

1. Imperfectly premixed flame

\[
\begin{align*}
\frac{\partial \xi}{\partial t} &= u - v \frac{\partial \xi}{\partial r} - S_u \sqrt{1 + \left( \frac{\partial \xi}{\partial r} \right)^2} \\
\frac{\partial \zeta}{\partial t} &= u - v \frac{\partial \zeta}{\partial r} + S_u \sqrt{1 + \left( \frac{\partial \zeta}{\partial r} \right)^2}
\end{align*}
\]

Equivalence/Fuel-air ratio $\phi$

\[S_u(\phi) = k_1 \phi^k e^{-k_3(\phi-k_4)^2}\]

(G. Abu-orf, 1996)[1]
2. Constant and Time-varying time-delays

\[
\begin{align*}
\phi(r, t) &= \phi_0(t - \bar{\tau}(r)) \\
\phi_0 &= \frac{|u|}{|\bar{u}|} \frac{L + \bar{\xi}}{L + \bar{\xi}}
\end{align*}
\]

where

\[
\bar{\tau}(r) = \frac{L + \bar{\xi}}{|\bar{u}|}
\]

\[
L + \xi(r, t) = \int_{t-\tau(r,t)}^{t} u(t')dt'
\]

\[
\Rightarrow \frac{\partial \tau}{\partial t}(r, t) = 1 - \frac{u(t) - \partial \xi / \partial t}{u(t - \tau(r, t))}
\]
Flame dynamics

\[ u = A \sin(2\pi f_i t) \]

\[ A = 0.1, f_i = 300 \text{ Hz} \]

\[ A = 0.3, f_i = 300 \text{ Hz} \]
Derive a linear flame model $G_f(s)$, which is the transfer function from the incoming velocity perturbation $\hat{u}$ to global heat release variation $\hat{Q}$.

**Fig.** Fully Premixed Flames

**Fig.** Imperfectly Premixed Flames
However the high frequency gain is seen to be too high relative to the experimental results. Introducing some mixing gives better match.
1. Helmholtz resonator

\[ \hat{u}_1 = G_h(s) \hat{p}_1 \]

2. Connect HR with downstream tube (Wave-based method)

\[ \frac{\hat{u}_1}{\hat{u}_1} = G_{ac}(s) \frac{\hat{Q}}{Q} \]

**Fig.** Simplified diagram of the duct geometry and flow
Fig. The frequency response of the acoustic model with short tube ($l_b = 80 \text{mm}$) and long tube ($l_b = 350 \text{mm}$)
Fig. Simulations of combustion instabilities: The fractional perturbations in $u_G(t)$ (the fluid velocity), $p_1(t)$ (the pressure), and $Q(t)$ (heat release).
Controller designed for linearized model with discretized pde using $\mathcal{H}_\infty$ techniques.

**Fig.** Simulation of stabilization of combustion instabilities: The fractional perturbations in $u_G(t)$ (the fluid velocity), $p_1(t)$ (the pressure), and control effort.
Analysis Tools - Integral Quadratic Constraints (Megretski and Rantzer)

- Approximate bounds on nonlinear deviations from linearized but discretized model.
- With $N$ discrete points for $r$ then we have a $Δ$ block of dimension $12N \times 6N$.
- Performance in both speed and robustness of the current LMI solvers were challenged by this scale of problem.
- Similar tools for delay approximation.
- Convergence as $N$ becomes large?
A Fault Detection Problem (Andras Varga, DLR)

\[ y = G_u u + G_d d + G_w w + G_f f \]

where
- \( y \) - output (measured)
- \( u \) - input (measured)
- \( d \) - disturbance (not measured)
- \( w \) - noise (not measured)
- \( f \) - fault (not measured)

Approx. Fault Detection Problem (AFDP): Find \( R \)

\[ r = R \begin{bmatrix} y \\ u \end{bmatrix} = R_u u + R_d d + R_w w + R_f f \]

such that \( R_u = 0 \) and \( R_d = 0 \) and (i) \( \| R_w \|_{2/\infty} \leq \gamma \), (ii) \( \| R_{fj} \|_{2/\infty} \geq \beta \) \( \forall j \).
Standard manipulations reduce the problem to finding $Q \in \mathcal{H}_{\infty}$ such that

(i) $\|QM_{wo}\|_{2/\infty} \leq \gamma$

(ii) $\|QN_{f,j}\|_{2/\infty} \geq \beta \quad \forall j$

where $M_{wo}$ is an outer function.

Want the maximize $\beta/\gamma$ the sensitivity to fault versus noise.

If $\|.\|_{\infty}$ in (i) then

- if $M_{wo}^{-1} \in \mathcal{H}_{\infty}$ then optimal solution is $Q = \gamma M_{wo}^{-1}$.
- otherwise can typically make $\beta$ arbitrarily large as "$Q \to M_{wo}^{-1}$".
Optimal solution with fixed form $Q = D + C(\sigma I - A)^{-1}B$ with $(A, B)$ given but $(C, D)$ to be chosen?
Let $X = [C \quad D]' [C \quad D]$ then

- $\|QM_{wo}\|_\infty \leq \gamma$ is equivalent to an LMI in $X$.
- $\|QN_{fj}\|_2 \geq \beta$ is equivalent to an LMI in $X$.
- but $\|QN_{fj}\|_\infty \geq \beta$ is NOT equivalent to an LMI.

[Note that although there appears to be a rank condition on $X$ a subsequent spectral factorization can remove this]
Question: is the problem with \[ \|QM_{wo}\|_\infty / \|QN_{fj}\|_\infty \] over \([C \ D]\) inherently non-convex?

Example: \(Q\) a \(2 \times 2\) constant. Let \(X = Q'Q\).

(i) \[ X \leq \begin{bmatrix} \frac{1+\alpha \omega^2}{1+\omega^2} & 0 \\ 0 & \frac{\alpha + \omega^2}{1+\omega^2} \end{bmatrix} \quad \forall \omega. \]

(ii) \[ \begin{bmatrix} \frac{1+j\omega}{1-j\omega} & 1 \end{bmatrix} X \begin{bmatrix} \frac{1-j\omega}{1+j\omega} \\ 1 \end{bmatrix} \geq \beta \quad \text{for some} \ \omega. \]

(iii) \[ \begin{bmatrix} 1 & -\frac{1+j\omega}{1-j\omega} \\ \frac{1+j\omega}{1-j\omega} & 1 \end{bmatrix} \geq \beta \quad \text{for some} \ \omega. \]

Some manipulation gives \(\beta_{\text{max}} = \frac{4\alpha}{1+\alpha}\) and \(X_{\text{opt}} = \frac{\alpha}{1+\alpha} \begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix} \).

Two disjoint sets of feasible solutions for \(\beta < \beta_{\text{max}}\).
Mathematics and Engineering as practiced at UCSD 2010 and beyond
G. Abu-orf.
*Laminal flamelet reaction rate modeling for spark ignition engines.*

A. M. Annaswamy and A. F. Ghoniem.
*Active control of combustion instabilities: theory and practice.*

R. Balachandran.
*Experimental investigation of the response of turbulent premixed flames to acoustic oscillations.*

G. J. Bloxsidge, A. P. Dowling, and P. J. Langhorne.
*Reheat buzz: An acoustically coupled combustion instability, part 2. theory.*

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*A kinematic model of a ducted flame.*

A. P. Dowling and S. Hubbard.
*Instability in lean premixed combustors.*
A. P. Dowling and A. S. Morgans.
Feedback control of combustion oscillations.

Response of a laminar premixed flame to flow oscillations: A kinematic model and thermoacoustic instability results.

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Reduced order kinematic modeling of ducted flames.