Problem 1. (16 pts.) Velocity $v(t)$ of a car, measured in ft / sec, increases according to the table below. To approximate the total distance traveled find:

(a) An upper estimate  

(b) A lower estimate 

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(t)$</td>
<td>0</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

How frequently must the velocity be recorded in order to estimate the total distance traveled with precision up to 24 ft?

Solution

upper estimate = right estimate = $(10 + 15 + 20 + 30) \cdot 2 = 75 \cdot 2 = 150$

lower estimate = left estimate = $(0 + 10 + 15 + 20) \cdot 2 = 45 \cdot 2 = 90$

If we record the velocity $n$ times during the 8 seconds to have the estimation of the total distance traveled with precision up to 24 ft then:

$$\left| (v(8) - v(0)) \frac{8 - 0}{n} \right| = (30 - 0) \frac{8}{n} \leq 24$$

Hence $\frac{240}{n} \leq 24$ which is equivalent to $10 \leq n$.

The velocity must be recorded at least 10 times to have the estimate of the total distance traveled with precision up to 24 ft.

Problem 2. (14 pts.)

Find the following derivative:

$$\frac{d}{dx} \int_1^{2x^2+1} \ln(2 + t^4) \, dt$$

Solution

Put $F(x) := \int_1^x \ln(2 + t^4) \, dt$. Hence by FTC

$$\frac{d}{dx} \int_1^{2x^2+1} \ln(2 + t^4) \, dt = (F(2x^2 + 1))' = F'(2x^2 + 1) \cdot 4x =$$

$$= 4x \ln(2 + (2x^2 + 1)^4).$$
Problem 3. (14 pts.) Find the area between the $x$-axis and the graph of $y = \cos x$ over the interval $[0, \pi/2]$.

**Solution**

The area = $\int_{0}^{\pi/2} \cos x \, dx = [\sin x]_{0}^{\pi/2} = \sin(\pi/2) - \sin 0 = 1 - 0 = 1$

Problem 4. (16 pts.) A stone is dropped from a top of a building. It hits the ground after 4 sec. What is the height of the building. Recall that: $s(t) = s_0 + v_0 t - \frac{1}{2} gt^2$, where $g = 32 \text{ ft/sec}^2$.

**Solution**

Observe that: $v_0 = 0$ and $s_0$ is the height of the building. Hence $0 = s(4) = s_0 - 16 \cdot 4^2 = s_0 - 256$. So the height of the building is:

$s_0 = 256 \text{ ft.}$
Problem 5. (24 pts.) Find the following integrals:

(a) $\int \cos x \ e^{5+\sin x} \, dx$

(b) $\int e^x \cos x \, dx$

Solution

(a) Integration by substitution. Take $u = 5 + \sin x$ then $du = \cos x \, dx$.

$$\int \cos x \ e^{5+\sin x} \, dx = \int e^u \, du = e^u + C = e^{5+\sin x} + C$$

(b) Integration by parts twice.

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx = \frac{1}{2}(e^x \sin x + e^x \cos x) + C$$

Problem 6. (16 pts) Find the following integral:

$$\int \frac{6}{(x-4)(x+2)} \, dx$$

Solution

$$\frac{6}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2} = \frac{(A+B)x + 2A - 4B}{(x-4)(x+2)}.$$  
From this $A+B = 0$ and $2A-4B = 6$. Hence $B = -A$ and $2A+4A = 6A = 6$. So $A = 1$ and $B = -1$. We have:

$$\int \frac{6}{(x-4)(x+2)} \, dx = \int \frac{1}{x-4} \, dx - \int \frac{1}{x+2} \, dx =$$

$$= \ln |x-4| - \ln |x+2| = \ln \left|\frac{x-4}{x+2}\right| + C.$$