FORM A

Problem 1. (12 pts.) Compute partial derivatives \( \frac{\partial f}{\partial x} \), \( \frac{\partial^2 f}{\partial x \partial y} \), \( \frac{\partial^2 f}{\partial x^2} \) of the following function:
\[
f(x, y) = \sin(x^2 + y^2)
\]

Problem 2. (14 pts.)
Use the linear approximation to estimate the given value:
\[
\sqrt{4.02^2 + 2.95^2}
\]
Problem 3. (12 pts.)
Let $f(x, y, z) = xy^{-1}e^{x+z}$ and let $r(t) = (e^t, t, 1+t)$. Compute the derivative $\frac{d}{dt}(f(r(t)))$.

Problem 4. (14 pts.)
Let $f(x, y, z) = x^2 y \sin (y + z)$ and let $x = u + v, y = uv, z = u - v$. Let $F(u, v) = f(x(u, v), y(u, v), z(u, v))$. Compute the partial derivatives:
\[ \frac{\partial F}{\partial u} \quad \text{and} \quad \frac{\partial F}{\partial v}. \]
Problem 5. (16 pts.)
Find the critical points of the following function and determine if they are local minima or maxima.
\[ f(x, y) = x^3 + 2xy - 2y^2 - 10x + 5 \]

Problem 6. (16 pts) Let \( f(x, y) = xy \). Find the minimum and maximum of the function \( f \) subject to the constrain \( x^2 + y^2 = 8 \).
Problem 7. (16 pts) Compute the following integral:

$$\int \int_D 3e^{y^3} \, dA$$

where $D$ is the region: $0 \leq x \leq y^2$, $0 \leq y \leq 1$. 