

Evaluate the following integral by converting it into polar coordinates:

$$\iint_D e^{-(x^2+y^2)/2} dx dy,$$

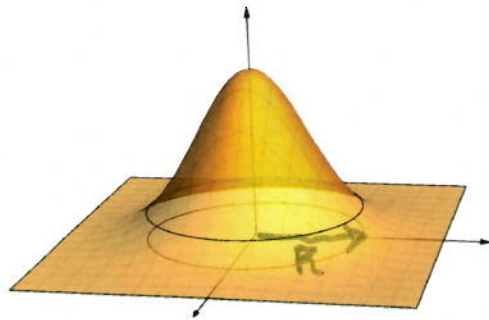
where D is a disk of radius R centered at the origin.

Change of coordinates:

$$r^2 = x^2 + y^2$$

$$dx dy = r \cdot dr d\theta$$

Disk D : $0 \leq r \leq R$
 $0 \leq \theta \leq 2\pi$



$$\int_{\theta=0}^{2\pi} \left(\int_{r=0}^R r \cdot e^{-r^2/2} dr \right) d\theta$$

$$\int_{\theta=0}^{2\pi} \left(-e^{-r^2/2} \Big|_0^R \right) d\theta$$

$$\int_{\theta=0}^{2\pi} (-e^{-R^2/2} + 1) d\theta$$

$$= 2\pi (-e^{-R^2/2} + 1)$$

$$\rightarrow 2\pi \cdot 1 \quad \text{as } R \rightarrow \infty$$