Math 180A Homework 2

Fall 2021

Due date: **11:59pm** (Pacific Time) on **Mon. Oct, 11** (via Gradescope)

In the "collaborators" field in Gradescope, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook and your notes. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Section 1 (short answer)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1 (numerical answer)**.** Suppose that $P(A|B) = 0.6$ and $P(B) = 0.5$. What is $P(A^c \cap B)$? Write your answer in decimal form.

Problem 2 (numerical answers)**.** You flip a fair coin three times.

(a) What is the probability that the third flip is heads given that the first two flips are heads?

(b) What is the probability that all three flips are heads given that at least two flips are heads? (Unlike part (a), you are not told *which* two of the flips are heads!)

Problem 3 (numerical answers — fraction)**.** You roll a fair six-sided die. Let A be the event that the roll is even, let B be the event that the roll is 1, 2, or 3, and let C be the event that the roll is 4 or 5.

(a) What is $P(A|B \cup C)$?

(b) What is $P(A|B) + P(A|C)$?

Express your answers as fractions in lowest terms.

Section 2 (long answer)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope (either typed or scanned from handwritten work). You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 4 (ASV Ex. 2.7a)**.** Let A and B be events in a probability space (Ω, P) with $0 < P(B) < 1$. Use the definition of conditional probability and the additivity of probability to show that $P(A^c|B) = 1 - P(A|B)$.

Problem 5. You flip a fair coin. If the flip is heads, then you roll a four-sided die. If the flip is tails, then you roll a six-sided die.

(a) Write a sample space Ω and probability P to describe the *whole* experiment.

(b) What is the probability that the die roll will be 1 or 2?

Problem 6. The Brooklyn Nets are playing basketball against the Milwaukee Bucks. There are ten seconds left in the game, and the Bucks are ahead by one point, so the Nets need two points to win. The Nets send Kevin Durant to take a final shot. Three things could happen:

- If Durant makes the shot, he will get two points.[1]
- If Durant misses the shot and does not get fouled, he will get zero points.
- If Durant misses the shot and gets fouled, he will have the opportunity to shoot two free throws. Each free throw is worth 1 point if he makes it.

We make some estimates based on past games:

- The probability of Durant making the first shot is 50%.
- The probability of Durant missing the shot and not getting fouled is 25%.
- If Durant misses and gets fouled, the probability of making each free throw is 88% (and they are independent), so the probability of making both free throws is $88\% \times 88\% = 77.44\%$, which we'll round to 77%.

What is the probability that Durant will score two points?

Problem 7. The wily Sheriff of Nottingham wants to catch the outlaw Robin Hood and his Merry Men. They are excellent archers, so he decides to hold an archery contest to figure out who they are.

Each contestant will shoot several arrows. On each shot, the Merry Men hit the bullseye with probability 9/10. There will also be villagers at the contest, who hit the bullseye with probability 1/10 on each shot. Suppose that, of the contestants who show up, 1/4 are Merry Men and 3/4 are Villagers.

(a) If a randomly chosen contestant hits the bullseye on his first shot, what is the probability that he is a Merry Man?

(b) A contestant is chosen randomly. What is the probability that he will miss his second shot given that he misses the first shot?

Problem 8 (Bonus – NOT TO BE TURNED IN)**.** Give one example of a statistic in real life that is a conditional probability that wasn't covered in lecture or discussion. Explain why it represents a conditional probability.

[1] Technically, it is possible Durant will make the shot *and* get fouled, and thus have the chance to shoot one free throw. However, this will not affect the outcome of the game since he has already scored two points. Hence, we do not need to separate out this case for the problem.