

# Math 180A: Introduction to Probability

## Quiz 2

Fall 2021

- You will have **50 minutes** to complete this quiz.
- Please have your student ID easily accessible to show to a proctor when asked.
- You may use one 8.5 x 11 inch sheet of handwritten notes, but no calculators, phones, or other study aids are permitted.
- Unless stated otherwise, if a question calls for a numerical answer, you don't need to simplify. **(For example, it's okay to write something like  $(0.9 - 0.01)7!/\binom{3}{2}$  as your answer.)**
- Please show your work and explain your answers for each problem unless otherwise specified—we will not award full credit for the correct numerical answer without proper explanation.
- Please write your final answer for each problem in the indicated area. If you do any work on the backs of the pages or on additional scratch paper that you would like to have graded, **please indicate that clearly; otherwise it will not be graded.**
- Don't forget to write your name on the top of every page.
- Good luck!

Name: \_\_\_\_\_

PID: \_\_\_\_\_

Seat Number: \_\_\_\_\_

*Just for fun: were you assigned the same seat as for Quiz 1?*

Yes    No    Unsure

Name: \_\_\_\_\_

**Problem 1: (1 points)** If  $X$  is a random variable with  $E(X) = 2$ , what is  $E(2X - 1)$ ?  
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- 3
- 4
- 8
- Not enough information

**Problem 2: (1 points)** Let  $X$  be the outcome of a single fair die roll. What is  $p_X(5)$ ?  
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- 3.5
- 5/6
- 1/6
- Not enough information

**Problem 3: (3 points)** If  $A$ ,  $B$ , and  $C$  are mutually independent events with  $P(A) > 0$  and  $P(B) > 0$  and  $P(C) > 0$ , which of the following are always true?  
(You do not need to show your work or justify your answers for this problem.)

Choose ALL that apply:

- $P(A \cap B) = P(A) \cdot P(B)$
- $P(A \cup B^c) = P(A) + P(B^c)$
- $P(A|B \cap C) = P(A)$

Name: \_\_\_\_\_

**Problem 4: (7 points)** You decide you want to build a spam filter to delete some of your many spam emails. You make the following observations:

- **50%** of all your emails are spam
- **20%** of your spam emails contain the phrase “CONSTRUCTION ALERT”
- Only **.1%** of your non-spam emails contain the phrase “CONSTRUCTION ALERT”

Let  $S$  be the event that a randomly selected email is spam, and let  $C$  be the event that it contains the phrase “CONSTRUCTION ALERT” So your observations can be written as:  $P(S) = .5$  and  $P(C|S) = .2$  and  $P(C|S^c) = .001$ . Find the following probabilities.

(a) (3 points)  $P(C)$  (the overall probability that an email will contain the phrase “CONSTRUCTION ALERT”)

Answer:

(b) (4 points)  $P(S|C)$  (the probability that an email is spam given that it contains the phrase “CONSTRUCTION ALERT”)

Answer:

Name: \_\_\_\_\_

**Problem 5: (8 points)** Let  $X$  be a random variable that takes the values  $-2$ ,  $0$ ,  $2$ , and  $3$  (and no other values), with the following probabilities:

$$P(X = -2) = 1/2$$

$$P(X = 0) = 1/12$$

$$P(X = 2) = 1/4$$

$$P(X = 3) = 1/6$$

(a) (2 points) Find  $E(X)$ .

Answer:

(b) (4 points) Let  $Y = X^2$ . Find the probability mass function of  $Y$ .

Answer:

Name: \_\_\_\_\_

**Problem 5 (continued)**

*(Copied from the previous page for reference)*

$$P(X = -2) = 1/2$$

$$P(X = 0) = 1/12$$

$$P(X = 2) = 1/4$$

$$P(X = 3) = 1/6$$

(c) (2 points) Find  $F_Y(1)$ , where  $Y = X^2$ .

Answer:

Name: \_\_\_\_\_

**Problem 6: (2 points)** *Bonus problem – I recommend focusing on the other problems and trying this one only if you have extra time!*

A jar contains 19 pink jelly beans and 100 yellow jelly beans. You randomly draw jelly beans one at a time from the jar **without replacement** until you get a pink jelly bean (the pink ones are your favorite). What is the expected number of yellow jelly beans you draw before the first pink one?

Answer: