Math 180A: Introduction to Probability
Quiz 3

Fall 2021

- You will have **50 minutes** to complete this quiz.
- Please have your student ID easily accessible to show to a proctor when asked.
- You may use one 8.5 x 11 inch sheet of handwritten notes, but no calculators, phones, or other study aids are permitted.
- Unless stated otherwise, if a question calls for a numerical answer, you don't need to simplify. **(For example, it's okay to write something like $(\mathbf{0.9-0.01})\mathbf{7!}/\binom{\mathbf{3}}{\mathbf{2}}$ as your answer.)**
- Please show your work and explain your answers for each problem unless otherwise specified– we will not award full credit for the correct numerical answer without proper explanation.
- Please write your final answer for each problem in the indicated area. If you do any work on the backs of the pages or on additional scratch paper that you would like to have graded, **please indicate that clearly; otherwise it will not be graded**.
- Don't forget to write your name on the top of every page.
- Good luck!

Name:____

PID:____________________

Seat Number:__________

Name: ______________________________

Problem 1: (2 points) Let X be a continuous random variable with the following probability density function:

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad \text{for all } x \in (-\infty, \infty).$$

Which of the following is equal to $P(|X| \geq 1)$?

(You do not need to show your work or justify your answers for this problem.)

Choose one:

- ○ $\displaystyle\int_1^{\infty} \frac{1}{\pi(1+x^2)}\,dx$
- ○ $\displaystyle\int_{-\infty}^{-1} \frac{1}{\pi(1+x^2)}\,dx + \int_1^{\infty} \frac{1}{\pi(1+x^2)}\,dx$
- ○ $\displaystyle\int_{-\infty}^{\infty} |x| \cdot \frac{1}{\pi(1+x^2)}\,dx$

Problem 2: (2 points) Suppose that the average score in a soccer game in the Premier League is 2.69 goals, and that scores follow a Poisson distribution. If there are 380 total games per season, on average how many games in a season will have 0 goals?

(You do not need to show your work or justify your answers for this problem.)

Choose one:

- ○ $380 \cdot e^{-2.69}\,\frac{(2.69)^0}{0!}$
- ○ $\left(e^{-2.69}\,\frac{(2.69)^0}{0!}\right)^{380}$
- ○ $e^{-380}\,\frac{(380)^0}{0!}$
- ○ $e^{-2.69}\,\frac{(2.69)^0}{0!}$

Name:

Problem 3: (4 points) In each election in a certain county, the county clerk is responsible for determining whether candidates from Party A or from Party B will appear first on the ballot (the party appearing first tends to have a small advantage in the vote). For each election, he chooses uniformly at random which of the two parties will be listed first. Suppose that out of 41 elections, Party A appears first on the ballot 40 times.[1]

If the county clerk is not cheating, and did indeed choose uniformly at random each time, what is the probability that Party A would appear first **40 or more times** out of 41 total?

Answer:

[1]This actually happened – you can search online for "Nicholas Caputo, Essex County, New Jersey"

Name:

Problem 4: (6 points) Dr. Gwen decides tonight is the night she will beat the Hotel Ghosts level of Super Mario Sunshine, no matter how long it takes her. Suppose that on each attempt she beats the level with probability $\frac{1}{50}$ (independently of all other attempts). Suppose also that it takes her 10 minutes to turn on and start the game (not including the time to make the first attempt), and that each attempt takes her 5 minutes. Let T be the total amount of time it takes her (in minutes) to beat the level.

(a) (4 points) Find the probability she beats the level in 1 hour or less.

Answer:

(b) (2 points) Find the standard deviation of T.

Answer:

Name:

Problem 5: (10 points) Let X be a (continuous) uniform random variable on the interval $[0, 2]$.

(a) (3 points) Find the cumulative distribution function $F_X(t)$.

Answer:

(b) (2 points) Find $E\left(\cos(X^2)\right)$. *(You can leave your answer as an unevaluated integral.)*

Answer:

Name:

Problem 5 (continued)

(c) (5 points) Define a random variable Z as follows. You flip a fair coin; if the flip is heads, you choose Z uniformly at random from the interval $[0, 1]$, and if the flip is tails, you choose Z uniformly at random from the interval $[0, 2]$. Find the probability density function $f_Z(x)$.

Answer: