Math 186, Winter 2005, Prof. Tesler – January 21, 2005
Binomial distribution

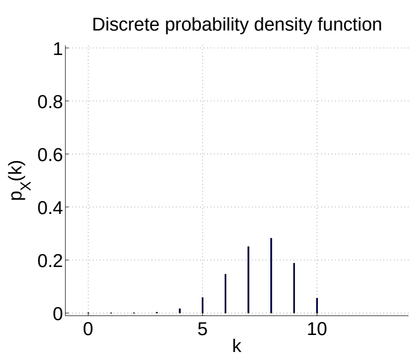
Here are the tabulated values and graphs of the discrete probability density function (pdf) and cumulative distribution function (cdf) for the binomial distribution with parameters $n = 10$ and $p = .75$.

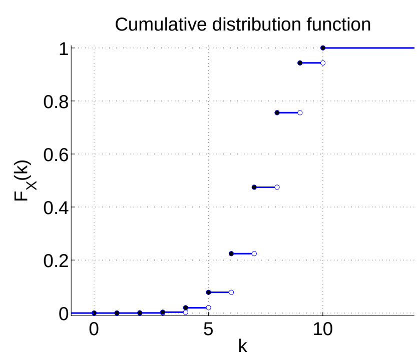
$$p_X(k) = P(X = k) = \begin{cases} \binom{10}{k}(.75)^k(.25)^{n-k} & \text{if } k = 0, 1, \ldots, 10; \\ 0 & \text{otherwise.} \end{cases}$$

$$F_X(k) = P(X \le k) = \begin{cases} 0 & \text{if } k < 0; \\ \sum_{r=0}^{\lfloor k \rfloor} \binom{10}{r}(.75)^r(.25)^{n-r} & \text{if } 0 \le k \le 10; \\ 1 & \text{if } k \ge 10. \end{cases}$$

Note: $\lfloor x \rfloor$ is the "floor" function (greatest integer $\le x$), which you may have seen written $[x]$ elsewhere: any real number x can be written uniquely as $x = m + \delta$, where m is an integer and δ is a real number with $0 \le \delta < 1$, and the floor of x is defined as $\lfloor x \rfloor = m$. For example, $\lfloor 3 \rfloor = 3$, $\lfloor -3 \rfloor = -3$, $\lfloor 3.2 \rfloor = 3$, $\lfloor -3.2 \rfloor = -4$.

k	pdf $p_X(k)$		cdf $F_X(k)$
		$k < 0$	0
0	0.00000095	$0 \le k < 1$	0.00000095
1	0.00002861	$1 \le k < 2$	0.00002956
2	0.00038624	$2 \le k < 3$	0.00041580
3	0.00308990	$3 \le k < 4$	0.00350571
4	0.01622200	$4 \le k < 5$	0.01972771
5	0.05839920	$5 \le k < 6$	0.07812691
6	0.14599800	$6 \le k < 7$	0.22412491
7	0.25028229	$7 \le k < 8$	0.47440720
8	0.28156757	$8 \le k < 9$	0.75597477
9	0.18771172	$9 \le k < 10$	0.94368649
10	0.05631351	$10 \le k$	1.00000000
other	0		





Sample uses of tables:

$$\begin{aligned}
P(X \le -3.2) &= 0 \\
P(X \le 12.8) &= 1 \\
P(X \le 6.5) &= F_X(6.5) = 0.22412491 && P(X = 6.5) = p_X(6.5) = 0 \\
P(X \le 6) &= F_X(6) = 0.22412491 && P(X = 6) = p_X(6) = 0.14599800 \\
P(X < 6) &= F_X(6^-) = 0.07812691 && \text{(Convert } P(X < a) \text{ into "}P(X \le a^-)\text{"} = F_X(a^-)) \\
P(X > 6) &= 1 - P(X \le 6) = 1 - F_X(6) = 1 - 0.22412491 = 0.77587508
\end{aligned}$$

$$\begin{aligned}
P(4 < X \le 8) &= P(X \le 8) - P(X \le 4) && \text{(Note: } X \le 4 \text{ is contained in the event } X \le 8) \\
&= F_X(8) - F_X(4) = 0.75597477 - 0.01972771 = 0.55869767
\end{aligned}$$

$$\begin{aligned}
P(4 \le X \le 8) &= \text{"}P(4^- < X \le 8)\text{"} = F_X(8) - F_X(4^-) = 0.75597477 - 0.00350571 = 0.75246906 \\
P(4 < X < 8) &= \text{"}P(4 < X \le 8^-)\text{"} = F_X(8^-) - F_X(4) = .47440720 - 0.01972771 = .45467949 \\
P(4 \le X < 8) &= \text{"}P(4^- < X \le 8^-)\text{"} = F_X(8^-) - F_X(4^-) \\
&= 0.47440720 - 0.00350571 = 0.47090149
\end{aligned}$$

An alternate way to compute these is to take advantage of the discrete values being integers; instead of using "a^-" we can go down to $a - 1$: $P(X < 6) = P(X \le 5) = F_X(5) = 0.07812691$.