Problem H-10. We will compute the mean of the geometric distribution. (Note: It’s also possible to compute $E(X^2)$ and then $\text{Var}(X) = E(X^2) - (E(X))^2$ by steps similar to these.)

(a) Set $q = 1 - p$. Show that

$$E(X) = \sum_{k=1}^{\infty} k \cdot q^{k-1} \cdot p$$

(b) Show that the above summation can be rewritten as follows:

$$E(X) = p \cdot \frac{\partial}{\partial q} \sum_{k=1}^{\infty} q^k$$

(c) The sum in part (b) is a geometric series. Evaluate the geometric series; replace the sum in (b) by this value; and do the derivative $\frac{\partial}{\partial q}$. The final answer will be a quotient of polynomials involving $p$ and $q$; there will not be an infinite sum remaining.

(d) Plug in $q = 1 - p$ and simplify to get the final answer.

Problem H-11. This problem deals with Haldane’s model of crossovers: assume crossovers are a Poisson process with rate $\lambda = 1 \text{ M}^{-1} = 0.01 \text{ cM}^{-1}$. Since this is a rate, the Poisson parameter is $\lambda d$.

(a) Two genes are on the same chromosome, 350 cM apart. Make a table of the probabilities of $k$ crossovers occurring in-between them for $k = -1, 0, 1, 2, 3, 4, 5$.

(b) For the genes in (a), what is the probability of 4 or more crossovers?

(c) What is the expected number of crossovers in-between the genes in (a), and what’s the standard deviation of that? What is the probability that the number of crossovers occurring will exactly equal the expected number?