

Math 186, Winter 2009, Prof. Tesler
Homework #7, Due Monday February 23, 2009

Larsen & Marx **Fourth Edition**:

4.2# 1, 4, 6, 11, 20, 27

~~4.3# 2, 10, 23, 26, 29~~ — **Section 4.3 is postponed.**

4.5# 3, 8*

and the problems below: H-11, H-12.

***Hints on the indicated problems:**

4.5.8. Ignore the hint in the book since it uses a technique we are not covering in class. Instead, use the fact that a negative binomial distribution can be written as a sum of independent geometric distributions with the same p . So a sum of independent negative binomial distributions with the same p can be turned into a sum of even more independent geometric distributions with the same p . That in turn can then be turned into a single new negative binomial distribution with a larger “ r ” parameter.

Problem H-11. This problem and the next one deal with Haldane’s model of crossovers: assume they are a Poisson process with rate $\lambda = 1 \text{ M}^{-1} = .01 \text{ cM}^{-1}$. Since this is a rate, the Poisson parameter is λd .

- (a) Two genes are on the same chromosome, 425 cM apart. Make a table of the probabilities of k crossovers occurring in-between them for $k = -1, 0, 1, 2, 3, 4, 5, 6$.
- (b) Again, two genes are on the same chromosome, 425 cM apart. What is the expected number of crossovers in-between them, and what’s the standard deviation of that? What is the probability that the number of crossovers occurring will exactly equal the expected number?

Problem H-12. During meiosis, two genes on the same chromosome are recombined if an odd number of crossovers occurs in-between them. Suppose that two genes are located a distance d apart, where d is measured in Morgans.

- (a) Show that the probability that the number of crossovers is odd equals $e^{-d} \sum_{k=0}^{\infty} \frac{d^{2k+1}}{(2k+1)!}$.
- (b) Show that $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = \frac{e^x - e^{-x}}{2}$ (which is called $\sinh(x)$) by using the Taylor series $e^x = \sum_{m=0}^{\infty} \frac{x^m}{m!}$.
- (c) Combine (a) and (b) to derive the formula given in homework 2, problem H-4: if genes A and B are a distance d apart (in Morgans) then the recombination rate between them is $r = \frac{1}{2}(1 - \exp(-2d))$.
- (d) When d is “small,” the distance d in Morgans is approximately the same as the recombination rate as a fraction. Now we will see why.
 - (i) Expand r as a Taylor series in d about $d = 0$ to at least three terms.
 - (ii) The first nonzero term in the Taylor series is d . Plot $r_1 = d$ and $r = (1 - \exp(-2d))/2$ as functions of d on the same graph. You may copy it from your graphing calculator or use a printout from a graphing program on a computer.
 - (iii) Examine the plots and give approximate values of d where r_1 and r are almost equal.