Math 186, Winter 2006, Prof. Tesler – March 10, 2006
"One sample" hypothesis tests for μ and σ

A normal distribution has mean μ and standard deviation σ, but the value of μ (and possibly σ) is not known to us! We must determine μ (and possibly σ) experimentally.

Let $y_1, \ldots, y_n$ be a random sample of size n from this normal distribution.

Let $\bar{y} = (y_1 + \ldots + y_n)/n$ be the sample mean and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{n\left(\sum_{i=1}^n {y_i}^2\right) - \left(\sum_{i=1}^n y_i\right)^2}{n(n-1)}$ be the sample variance.

We want to test $\bar{y}$ as an estimate of μ (is $\bar{y}$ close to a constant μ_0?) or s as an estimate of σ (is s close to a constant σ_0?) under various circumstances.

z-test: μ unknown, σ known; testing μ. See Chap. 5.3 and 6.2.

Note: z_α is defined so that $P(Z \ge z_\alpha) = \alpha$ in the standard normal distribution. Table A.1 gives $\Phi(z) = P(Z \le z)$ so solve $\Phi(z_\alpha) = 1 - \alpha$.

type	Hypotheses	Test Statistic: Z	Test H_0 at α sig. level: Reject H_0 when	$100(1-\alpha)\%$ conf. int. for μ under H_0
one-sided (right)	H_0: $\mu = \mu_0$ vs. H_1: $\mu > \mu_0$		$z \ge z_\alpha$	$\left(-\infty\ ,\ \bar{y} + z_\alpha \frac{\sigma}{\sqrt{n}}\right)$
one-sided (left)	H_0: $\mu = \mu_0$ vs. H_1: $\mu < \mu_0$	$z = \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}$	$z \le -z_\alpha$	$\left(\bar{y} - z_\alpha \frac{\sigma}{\sqrt{n}}\ ,\ \infty\right)$
two-sided	H_0: $\mu = \mu_0$ vs. H_1: $\mu \ne \mu_0$		$z \le -z_{\alpha/2}$ or $z \ge z_{\alpha/2}$	$\left(\bar{y} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\ ,\ \bar{y} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$

Student's t-test: μ unknown, σ unknown; testing μ. See Chap. 7.2–7.4

Note: $t_{\alpha,k}$ is defined so that $P(T_k \ge t_{\alpha,k}) = \alpha$ in the t distribution with k degrees of freedom. Table A.2.

type	Hypotheses	Test Statistic: T_{n-1}	Test H_0 at α sig. level: Reject H_0 when	$100(1-\alpha)\%$ conf. int. for μ under H_0
one-sided (right)	H_0: $\mu = \mu_0$ vs. H_1: $\mu > \mu_0$		$t \ge t_{\alpha,n-1}$	$\left(-\infty\ ,\ \bar{y} + t_{\alpha,n-1} \frac{s}{\sqrt{n}}\right)$
one-sided (left)	H_0: $\mu = \mu_0$ vs. H_1: $\mu < \mu_0$	$t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}}$	$t \le -t_{\alpha,n-1}$	$\left(\bar{y} - t_{\alpha,n-1} \frac{s}{\sqrt{n}}\ ,\ \infty\right)$
two-sided	H_0: $\mu = \mu_0$ vs. H_1: $\mu \ne \mu_0$		$t \le -t_{\alpha/2,n-1}$ or $t \ge t_{\alpha/2,n-1}$	$\left(\bar{y} - t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}\ ,\ \bar{y} + t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}\right)$

χ^2-test ("chi-squared test"): μ unknown, σ unknown; testing σ^2. See Chap. 7.5.

Note: $\chi^2_{\alpha,k}$ is defined so that $P(\chi^2_k \le \chi^2_{\alpha,k}) = \alpha$ in the χ^2 distribution with k degrees of freedom. This is backwards from $t_{\alpha,k}$. Table A.3.

type	Hypotheses	Test Statistic: χ^2_{n-1}	Test H_0 at α sig. level: Reject H_0 when	$100(1-\alpha)\%$ conf. int. for σ^2 under H_0
one-sided (right)	H_0: $\sigma^2 = {\sigma_0}^2$ vs. H_1: $\sigma^2 > {\sigma_0}^2$		$\chi^2 \ge \chi^2_{1-\alpha,n-1}$	$\left(0\ ,\ \frac{(n-1)s^2}{\chi^2_{\alpha,n-1}}\right)$
one-sided (left)	H_0: $\sigma^2 = {\sigma_0}^2$ vs. H_1: $\sigma^2 < {\sigma_0}^2$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2 \le \chi^2_{\alpha,n-1}$	$\left(\frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}\ ,\ \infty\right)$
two-sided	H_0: $\sigma^2 = {\sigma_0}^2$ vs. H_1: $\sigma^2 \ne {\sigma_0}^2$		$\chi^2 \le \chi^2_{\alpha/2,n-1}$ or $\chi^2 \ge \chi^2_{1-\alpha/2,n-1}$	$\left(\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\ ,\ \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}\right)$