Math 20C  Practice Final  Name: ________________________________
Profs. Eggers & Tesler  PID: ________________________________

Instructions:
• No calculators, tablets, phones, or other electronic devices are allowed during this exam.
• You may use one handwritten sheet of notes (8.5”×11” double-sided), but no books or other aids.
• Fill in your Name and PID on this page.
• Read each question carefully and answer each question completely.
• Write your solutions clearly and legibly.
• Show all of your work; no credit will be given for unsupported answers.
• Circle your final answers.
• Exams must be turned in promptly when time is called.

0. (1 point) Carefully read and complete the instructions on this exam and any additional instructions announced during the exam.

1. Evaluate the iterated integral \( \int_0^4 \int_{\sqrt{x}}^2 6 \cos(y^3) \, dy \, dx \).
   \text{Hint: Write it another way.}

2. Let \( \vec{a}, \vec{b}, \) and \( \vec{c} \) be nonzero vectors in \( \mathbb{R}^3 \). Set
   \[ \vec{v} = \vec{a} \times (\vec{b} \times \vec{c}) \quad \text{and} \quad \vec{w} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}. \]
   For each of the following, provide a brief explanation of no more than two or three sentences.
   (a) Explain why \( \vec{v} \) is orthogonal to \( \vec{a} \).
   (b) Explain why \( \vec{v} \) lies in the plane spanned by \( \vec{b} \) and \( \vec{c} \). That is, explain why \( \vec{v} \) is orthogonal to the normal vector of the plane parallel to both \( \vec{b} \) and \( \vec{c} \).
   (c) Explain why \( \vec{w} \) is also orthogonal to \( \vec{a} \).
   (d) Explain why \( \vec{w} \) also lies in the plane spanned by \( \vec{b} \) and \( \vec{c} \).
   (e) What is the relationship between \( \vec{v} \) and \( \vec{w} \), based on parts (a)–(d) above?

3. Let \( D = \{ (x, y) : x^2 + y^2 \leq 4 \} \). Briefly explain why \( \int_D \sin^2(x) \, dA \leq \pi \).
   \text{Note: do not attempt to evaluate the integral.}

4. Use cylindrical coordinates to compute the volume of the solid region \( W \) that lies above the surface \( z = x^2 + y^2 \) and below the surface \( z = 8 - x^2 - y^2 \).

5. The acceleration, initial velocity, and initial position of a particle traveling through space are given by \( \vec{a}(t) = (-6, 2, 4), \vec{v}(0) = (2, -5, 1), \) and \( \vec{r}(0) = (-3, 6, 2), \) respectively. The particle’s trajectory intersects the \( xz \)-plane exactly twice. Find these two intersection points.

6. Find an equation for the plane that contains the line \( \vec{\lambda}(t) = (-1, 1, 2) + t(3, 2, 4) \) and is perpendicular to the plane \( 2x + y - 3z = 4 \).

7. Evaluate the triple integral \( \iiint_W y \, dV \), where \( W \) is the region bounded by
   \[ z = 0, \quad x + y + z = 4, \quad y - x = 0, \quad x = 2, \quad \text{and} \quad y = 0. \]
8. For each point on the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), there is a corresponding inscribed rectangle with vertices \((x, y), (-x, y), (x, -y), (-x, -y)\). See the graphic below. Use the method of Lagrange multipliers to find the inscribed rectangle with maximum area.

9. The two lines \( \vec{r}_1(t) = \langle t, 5-3t, 5-2t \rangle \) and \( \vec{r}_2(t) = \langle 3+2t, 4+2t, 2-t \rangle \) intersect at a point. Find an equation for the plane that contains both of these lines.

10. The path \( \vec{c}(t) = \langle 3 \cos(t), 3 \sin(t), 2t^{3/2} \rangle \) traces out a curve \( C \).

   (a) Find the length of the curve \( C \) traced out by \( \vec{c}(t) \) for \( 0 \leq t \leq 3 \).

   (b) Give an example of a path \( \vec{r}(t) \) in three dimensions that has constant speed but nonzero acceleration. Be sure to show your example has constant speed and nonzero acceleration.