Here are the final answers and comments on what method to use, but not all details are shown. On the midterm, this would not be sufficient; you need to show all your work, not just the final answers.

(0) You need to follow all the instructions. This includes writing your name, PID, etc.

(1a) \(6\hat{i} - 6\hat{j} + 3\hat{k}\)

(1b) Compute the magnitude of the answer in (a): 9

(1c) Compute \(\vec{v} \times (\vec{v} \times \vec{w})\):

\[-9\hat{i} - 18\hat{j} - 18\hat{k}\]

Or compute \((\vec{v} \times \vec{w}) \times \vec{v}\):

\[9\hat{i} + 18\hat{j} + 18\hat{k}\]

Any nonzero multiple of these is also a correct answer, such as \(\hat{i} + 2\hat{j} + 2\hat{k}\).

**Warning:** The cross-product is not associative. Usually, \((\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})\), although there are special cases where those are equal. So it is invalid to use \(\vec{v} \times (\vec{v} \times \vec{w}) = (\vec{v} \times \vec{v}) \times \vec{w} = \vec{0}\). As shown above, this cross-product does not come out to \(\vec{0}\).

(2a) There are multiple correct answers. They should be in the form \(\vec{\ell}(t) = \vec{\ell}_0 + t\vec{v}\) where \(\vec{\ell}_0\) is the position vector of a point on the line (either \(P\) or \(Q\)) and \(\vec{v}\) is a direction vector along the line (either \(\overrightarrow{PQ}\) or \(\overrightarrow{QP}\)).

Using point \(P\) and direction vector \(\overrightarrow{PQ}\):

\(\vec{\ell}(t) = (0, 2, -1) + t(-3, -1, 1)\)

Using point \(Q\) and direction vector \(\overrightarrow{QP}\):

\(\vec{\ell}(t) = (-3, 1, 0) + t(3, 1, -1)\)

(2b) There are multiple correct answers.

The normal vector of the plane is the direction vector of the line, \((-3, -1, 1)\) (or its negative).

In vector notation: \((-3, -1, 1) \cdot (\vec{r} - (-3, 1, 0)) = 0\)

In scalar notation: \(-3(x - (-3)) + (-1)(y - 1) + 1(z - 0) = 0\), or \(-3(x + 3) - (y - 1) + z = 0\).

Expanding and simplifying gives \(-3x - 9 - y + 1 + z = 0\) so \(-3x - y + z - 8 = 0\) or \(-3x - y + z = 8\)

or anything equivalent to that, such as negating it or any nonzero multiple of it.

(3) At \((0, 0)\), the given function has the form \(0/0\), so we cannot use continuity to compute the limit.

Consider different lines through the origin, \(y = mx\), for different slopes \(m\). On such lines,

\[
\frac{2xy}{x^2 + y^2} = \frac{2x(mx)}{x^2 + (mx)^2} = \frac{2mx^2}{x^2 + m^2x^2} = \frac{2m}{1 + m^2}.
\]

Since this depends on \(m\), the limiting value as we approach the origin in different directions is different. Thus, the limit **does not exist**.
(4) Since $\|\vec{u} + \vec{v}\| = 2/3$, then $\|\vec{u} + \vec{v}\|^2 = 4/9$. We can also compute it like this:

\[
\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = (\vec{u} \cdot \vec{u}) + (\vec{v} \cdot \vec{v}) + (\vec{u} \cdot \vec{v}) + (\vec{v} \cdot \vec{v})
\]

Since $\vec{u}$ is a unit vector, $\|\vec{u}\| = 1$ so $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2 = 1^2 = 1$. Same with $\vec{v}$.

\[
\|\vec{u} + \vec{v}\|^2 = 1 + 2(\vec{u} \cdot \vec{v}) + 1 = 2 + 2(\vec{u} \cdot \vec{v})
\]

This gives $4/9 = 2 + 2\vec{u} \cdot \vec{v}$, so $\vec{u} \cdot \vec{v} = ((4/9) - 2)/2 = -7/9$.

In the same way, $\|\vec{u} - \vec{v}\|^2 = 2 - 2\vec{u} \cdot \vec{v} = 2 - 2(-7/9) = 32/9$.

Then $\|\vec{u} - \vec{v}\| = \sqrt{32/9} = \left(\frac{4}{3}\right)\sqrt{2}$.

(5) Try to solve $\vec{r}_1(t) = \vec{r}_2(u)$. Note that we rename the parameter of the second curve to $u$ instead of $t$, since the lines may reach the intersection point at different times.

\[
\langle -9, 6, -8 \rangle + t \langle 4, -1, 3 \rangle = \langle 7, 0, 2 \rangle + u \langle 2, -1, 1 \rangle
\]

\[
-9 + 4t = 7 + 2u \quad 6 - t = -u \quad -8 + 3t = 2 + u
\]

If there was not a solution, then the lines would not intersect. But you should find that $t = 2$, $u = -4$ solves these, so they do intersect. Then plug those values in to find where they intersect:

\[
\vec{r}_1(2) = \langle -9 + 4(2), 6 - 2, -8 + 3(2) \rangle = \langle -1, 4, -2 \rangle
\]

\[
\vec{r}_2(-4) = \langle 7 + 2(-4), 0 - (-4), 2 + (-4) \rangle = \langle -1, 4, -2 \rangle
\]

Both give the same value, so **the lines intersect at point $(-1, 4, -2)$**.

Note: Technically, $\vec{r}_1(t), \vec{r}_2(u)$ are position vectors of points. So in this notation, $\langle -1, 4, -2 \rangle$ is the position vector of the intersection point, while $(-1, 4, -2)$ is the intersection point. Our book uses parentheses for both vectors and points, so in the book’s notation, the position vector $(-1, 4, -2)$ and the point $(-1, 4, -2)$ would be written the same.