Problem H-1. A value is determined as follows by flipping a fair coin: heads=4, tails=6. The coin is flipped twice. Let $X$ denote the value of the first flip, and $Y$ denote the sum of the values of the two flips.

(a) Make a table giving the joint pdf of $X$ and $Y$.

(b) Use the table to compute the marginal densities $P_X(x)$, $P_Y(y)$.

(c) Compute $E(X)$, $E(Y)$, and $\text{Var}(X)$.

(d) (i) Compute $\text{Cov}(X,Y)$.
   (ii) Explain why $\text{Cov}(X,Y) = 0$ or $\neq 0$, whichever the case may be.
   (iii) Also explain the relation between the numerical values of $\text{Var}(X)$ and $\text{Cov}(X,Y)$.

(e) The cumulative distribution function (abbreviated cdf) of random variable $X$ is $F_X(x) = P(X \leq x)$, and for $Y$ it’s $F_Y(y) = P(Y \leq y)$. Compute the functions $F_X(x)$ and $F_Y(y)$ for all real numbers. (These will be piecewise functions with several cases.)

(f) The coin is flipped three times and the values sum to 14. What is the probability that the second flip is tails?

The problems below concern Mendel’s pea plant experiments. Two of the genes he studied were height (alleles $T=tall$, $t=short$) and pea shape (alleles $R=round$, $r=wrinkled$). $T$ and $R$ are dominant, so for example, the genotypes $RR$ and $Rr$ give a round phenotype and the genotype $rr$ gives a wrinkled phenotype. Genes $T$ and $R$ are on different chromosomes, so they recombine independently.

Problem H-2. A large quantity of seeds are produced from $Rr \times Rr$ crosses. The seeds are numbered randomly and planted. The offspring are examined in order of their assigned numbers to determine if their peas are round or wrinkled. Compute the expected number of plants examined until the eighth plant with round peas is observed. Also compute the standard deviation of this number. (E.g., if plants 1,2,3, . . . are seen to be round, round, wrinkled, round, round, round, wrinkled, . . . then the 3rd observation of a round plant is on observation number 5.)

Problem H-3. 12 independent offspring are produced from an $Rr \times Rr$ cross of pea plants.

(a) Compute the expected number of offspring with round peas.

(b) Compute the variance and the standard deviation of the quantity in (a).

(c) Compute the probability that the number of offspring with round peas is exactly the expected number computed in (a).

(d) Redo (a) and (c) for the case when there are 13 independent offspring instead of 12.

Problem H-4. Two pea plants with genotype $TtRr$ are crossed and yield 20 offspring. Use the multinomial distribution to determine the probability that 8 offspring are tall and round, 3 are tall and wrinkled, 7 are short and round, and 2 are short and wrinkled.