Problem H-1. A value is determined from flipping a fair coin as follows: heads=5, tails=10. The coin is flipped twice. Let $X$ denote the value of the first flip, and $Y$ denote the sum of the values of the two flips.

(a) Write out a table giving the joint pdf of $X$ and $Y$.
(b) Use the table to compute the marginal densities $P_X(x)$, $P_Y(y)$.
(c) Compute $E(X)$, $E(Y)$, and $\text{Var}(X)$.
(d) (i) Compute $\text{Cov}(X,Y)$.
(ii) Explain why $\text{Cov}(X,Y) = 0$ or $\neq 0$, whichever the case may be.
(iii) Also explain the relation between the numerical values of $\text{Var}(X)$ and $\text{Cov}(X,Y)$.
(e) The cumulative distribution function (abbreviated cdf) of random variable $X$ is $F_X(x) = P(X \leq x)$, and for $Y$ it’s $F_Y(y) = P(Y \leq y)$. Compute the functions $F_X(x)$ and $F_Y(y)$ for all real numbers. (These will be piecewise functions with several cases.)
(f) The coin is flipped three times and the values sum to 20. What is the probability that the second flip is tails?

Problem H-2. A large quantity of seeds are produced from $Rr \times Rr$ crosses, the seeds are numbered randomly, and planted. The offspring are examined in order of their assigned numbers to determine if their peas are round or wrinkled. Compute the expected number of plants that would have to be examined until the twelfth plant with round peas is observed. Also compute the standard deviation of this number. (E.g., if plants 1, 2, 3, . . . are seen to be round, round, wrinkled, round, round, round, wrinkled, . . . then the 3rd observation of a round plant is on observation number 5.)

Problem H-3. 20 independent offspring are produced from an $Rr \times Rr$ cross of pea plants.

(a) Compute the expected number of offspring with round peas.
(b) Compute the variance and the standard deviation of the quantity in (a).
(c) Compute the probability that the number of offspring with round peas is exactly the expected number computed in (a).
(d) Redo (a) and (c) for the case when there are 15 independent offspring instead of 20.

Problem H-4. Two pea plants with genotype $TtRr$ are crossed and yield 20 offspring. Use the multinomial distribution to determine the probability that 8 offspring are tall and round, 4 are tall and wrinkled, 6 are short and round, and 2 are short and wrinkled.