On Simultaneous Stabilizability with Local-Global Principle without Coprime Factorizability

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Abstract

This paper addresses the applicability of the local-global principle to the simultaneous stabilizability of plants. We show that the simultaneous stabilizability over the set of stable causal transfer functions cannot be given by the simultaneous stabilizability over local rings in general. We do not assume the coprime factorizability of given plants.

Keywords

Linear System, Simultaneous Stabilization, Coprime Factorization, Local-Global Principle, Coordinate-Free Approach.

1 Introduction

Using the local-global principle[1, 2] gives the stabilizability of plants and stabilizing controllers of plants without coprime factorizability[3, 4]. Parametrization of stabilizing controllers without coprime factorizability[5] is also given by using the local-global principle.

Simultaneous stabilization problem is a problem to stabilize two (or more) plants by a controller. When a plant admits a doubly coprime factorization, this problem can be reduced to a problem to find a
matrix (e.g. Theorem 5.4.2 of [6]). However, in the case where a plant does not admit a doubly coprime factorization or we do not know whether or not a plant admits a doubly coprime factorization, we cannot reduce the simultaneous stabilization problem to such a problem.

It is known that there are models such that some stabilizable plants do not admit a doubly coprime factorization [7]. Thus, the simultaneous stabilization problem without coprime factorizability comes now an interesting problem. Our interest is to obtain some criterion of solving the simultaneous stabilization problem without coprime factorizability (such as Theorem 5.4.2 of [6]).

Even though the local-global principle is a powerful tool, in this paper, we show that the simultaneous stabilizability over original ring, which is the set of stable causal transfer functions, cannot be given by the simultaneous stabilizabilities over local rings in general.

This paper is organized as follows. After this introduction, we begin on the introduction of the coordinate-free approach, we used in this paper, in Section 2, including definitions. In Section 3, we show how the local-global principle can be applied to the stabilizability and stabilization of plants. Then, in Section 4, we consider the simultaneous stabilizability with the local-global principle.

2 Coordinate-Free Approach

We start by giving the preliminary of the coordinate-free approach. In the following we introduce the notations used in this paper. Then we give the formulation of the feedback stabilization problem.

2.1 Notations

Commutative Rings  In this paper, we consider that any commutative ring has the identity 1 different from zero. Let \( R \) denote a (unspecified) commutative ring. The total ring of fractions of \( R \) is denoted by \( \mathcal{F}(R) \).

We will consider that the set of stable causal transfer functions is a commutative ring denoted by \( A \). From the sense of the transfer functions we consider that the commutative ring \( A \) satisfies the invariant
basis property (cf. [2]). In addition to $\mathcal{A}$, we will use the following three kinds of ring of fractions. The first one appears as the total ring of fractions of $\mathcal{A}$, which is denoted by $\mathcal{F}(\mathcal{A})$ or simply by $\mathcal{F}$; that is, $\mathcal{F} = \{n/d \mid n, d \in \mathcal{A}, \ d \text{ is a nonzerodivisor}\}$. This will be considered as the set of all possible transfer functions. The second one is associated with the set of powers of a nonzero element of $\mathcal{A}$. Let $f$ denote a nonzero element of $\mathcal{A}$. Given a set $S_f = \{f^n \mid n \in \mathcal{A}, \ d \in S_f\}$, which is a multiplicative subset of $\mathcal{A}$, we denote by $\mathcal{A}_f$ the ring of fractions of $\mathcal{A}$ with respect to the multiplicative subset $S_f$; that is, $\mathcal{A}_f = \{n/d \mid n \in \mathcal{A}, \ d \in S_f\}$. The last one is the total ring of fractions of $\mathcal{A}_f$, which is denoted by $\mathcal{F}(\mathcal{A}_f)$; that is, $\mathcal{F}(\mathcal{A}_f) = \{n/d \mid n, d \in \mathcal{A}_f, \ d \text{ is a nonzerodivisor of } \mathcal{A}_f\}$. If $f$ is a nonzerodivisor of $\mathcal{A}$, $\mathcal{F}(\mathcal{A}_f)$ coincides with the total ring of fractions of $\mathcal{A}$. Otherwise, they do not coincide.

Matrices The set of matrices over $\mathcal{R}$ of size $x \times y$ is denoted by $\mathcal{R}^{x \times y}$. Further, the set of square matrices over $\mathcal{R}$ of size $x$ is denoted by $(\mathcal{R})_x$. The identity and the zero matrices are denoted by $E_x$ and $O_{x \times y}$, respectively, if the sizes are required, otherwise they are denoted by $E$ and $O$.

Matrix $A$ over $\mathcal{R}$ is said to be nonsingular (singular) over $\mathcal{R}$ if the determinant of the matrix $A$ is a nonzerodivisor (a zerodivisor) of $\mathcal{R}$. Matrices $A$ and $B$ over $\mathcal{R}$ are right-coprime over $\mathcal{R}$ if there exist matrices $X$ and $Y$ over $\mathcal{R}$ such that $XA + YB = E$ holds. Further, an ordered pair $(N, D)$ of matrices $N$ and $D$ is said to be a right-coprime factorization over $\mathcal{R}$ of $P$ if (i) $D$ is nonsingular over $\mathcal{R}$, (ii) $P = ND^{-1}$ over $\mathcal{F}(\mathcal{R})$, and (iii) $N$ and $D$ are right-coprime over $\mathcal{R}$. As the parallel notion, the left-coprime over $\mathcal{R}$ and the left-coprime factorization over $\mathcal{R}$ of $P$ are defined analogously. If a plant has both a right- and a left-coprime factorizations over $\mathcal{R}$, then the plant is said to admit a doubly coprime factorization over $\mathcal{R}$. For short, we may omit “over $\mathcal{R}$” when $\mathcal{R} = \mathcal{A}$.

2.2 Feedback Stabilization Problem

The stabilization problem follows that of Desoer et al. of [8], Sule in [3], and Mori and Abe in [4], who consider the feedback system $\Sigma$ [6, Ch.5, Figure 5.1] as in Figure 1. For further details the reader is referred to [6]. The plant we consider has $m$ inputs and $n$ outputs, and its transfer matrix, which is
also called a plant itself simply, is denoted by $P$ and belongs to $\mathcal{F}^{n \times m}$. We can always represent $P$ in the form of a fraction $P = N D^{-1}$ ($P = \bar{D}^{-1} \bar{N}$), where $N \in \mathcal{A}^{n \times m}$ ($\bar{N} \in \mathcal{A}^{n \times m}$) and $D \in (\mathcal{A})_m$ ($\bar{D} \in (\mathcal{A})_n$) with nonsingular $D$ ($\bar{D}$).

**Definition 2.1** For $P \in \mathcal{F}^{n \times m}$ and $C \in \mathcal{F}^{m \times n}$, a matrix $H(P, C) \in (\mathcal{F})_{m+n}$ is defined as

$$
H(P, C) = \begin{bmatrix}
(E_n + PC)^{-1} & -P(E_m + CP)^{-1} \\
C(E_n + PC)^{-1} & (E_m + CP)^{-1}
\end{bmatrix}
$$

(1)

provided that $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$. This $H(P, C)$ is the transfer matrix from $[u_1^t \ u_2^t]^t$ to $[e_1^t \ e_2^t]^t$ of the feedback system $\Sigma$. If (i) $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$ and (ii) $H(P, C) \in (\mathcal{A})_{m+n}$, then we say that the plant $P$ is stabilizable, $P$ is stabilized by $C$, and $C$ is a stabilizing controller of $P$.

Since the transfer matrix $H(P, C)$ of the stable causal feedback system has all entries in $\mathcal{A}$, we call the above notion $\mathcal{A}$-stabilizability. One can further introduce the notion of $\mathcal{A}_f$-stabilizability as follows.

**Definition 2.2** Let $f$ be a nonzero element of $\mathcal{A}$. If (i) $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}_f$ and (ii) $H(P, C) \in (\mathcal{A}_f)_{m+n}$, then we say that the plant $P$ is $\mathcal{A}_f$-stabilizable, $P$ is $\mathcal{A}_f$-stabilized by $C$, and $C$ is an $\mathcal{A}_f$-stabilizing controller of $P$.

The causality of transfer functions is an important physical constraint.
Definition 2.3 (Definition 3.1 of [9]) Let $Z$ be a prime ideal of $A$, with $Z \neq A$, including all zerodivisors. Define the subsets $\mathcal{P}$ and $\mathcal{P}_S$ of $\mathcal{F}$ as follows:

$$\mathcal{P} = \{n/d \in \mathcal{F} | n \in A, \ d \in A\setminus Z\},$$

$$\mathcal{P}_S = \{n/d \in \mathcal{F} | n \in Z, \ d \in A\setminus Z\}.$$

Then every transfer function in $\mathcal{P}$ ($\mathcal{P}_S$) is called causal (strictly causal). Analogously, if every entry of a transfer matrix $F$ is in $\mathcal{P}$ ($\mathcal{P}_S$), the transfer matrix $F$ is called causal (strictly causal). A matrix over $A$ is said to be $Z$-nonsingular if the determinant is in $A\setminus Z$, and $Z$-singular otherwise.

Finally, we introduce the notion of simultaneous stabilization.

Definition 2.4 Let $P_0$ and $P_1$ be plants in $\mathcal{P}^{n \times m}$. If $C$ in $\mathcal{F}^{m \times n}$ is a stabilizing controller of $P_0$ and $P_1$, then $C$ is said to be a simultaneously stabilizing controller of $P_0$ and $P_1$, and that $P_0$ and $P_1$ are simultaneously stabilized by $C$. If there exists a simultaneously stabilizing controller of $P_0$ and $P_1$, then they are said to be simultaneously stabilizable.

For the case we consider $A_f$-stabilizability instead of $A$-stabilizability, we will use “simultaneously $A_f$-stabilizing controller,” “simultaneously $A_f$-stabilized,” and “simultaneously $A_f$-stabilizable” analogously.

3 Local-Global Principle for Stabilizability and Stabilization

In this section, we review the application of the local-global principle to the stabilizability and the stabilization of plants.

Theorem 3.1 Let $P$ be a plant in $\mathcal{P}^{n \times m}$. Then the following conditions are equivalent.

(i) $P$ is stabilizable.
(ii) there exists a finite subset \( \Lambda \) of \( \mathcal{A} \) such that (a) \( \sum_{\lambda \in \Lambda} \lambda = 1 \) and (b) for each \( \lambda \in \Lambda \), \( P \) admits a doubly coprime factorization over \( \mathcal{A}_{\lambda} \).

(ii) there exists a finite subset \( \Lambda \) of \( \mathcal{A} \) such that (a) \( \sum_{\lambda \in \Lambda} \lambda = 1 \) and (b) for each \( \lambda \in \Lambda \), \( P \) is \( \mathcal{A}_{\lambda} \)-stabilizable.

In the theorem, without loss of generality, the condition (a) can be replaced by “for each \( \lambda \in \Lambda \), there exists \( a_{\lambda} \in \mathcal{A} \) such that \( \sum_{\lambda \in \Lambda} a_{\lambda} \lambda = 1 \)”. But for the simplicity, we use the original condition.

**Proof.** For “(i)\( \Rightarrow \)(ii)”, the reader should refer to Section C of [5]. “(ii)\( \Rightarrow \)(iii)” is obvious.

In the following, we give the proof of “(iii)\( \Rightarrow \)(i).”

Suppose that (iii) holds. Let \( C_{\lambda} \) be an \( \mathcal{A}_{\lambda} \)-stabilizing controller of \( P \). Then \( H(P, C_{\lambda}) \) is over \( \mathcal{A}_{\lambda} \). For sufficiently large positive integer \( \mu \), we have \( \lambda^{\mu} H(P, C_{\lambda}) \) is over \( \mathcal{A} \) for each \( \lambda \in \Lambda \).

Suppose that \( \sum_{\lambda \in \Lambda} a_{\lambda} \lambda^{\mu} = 1 \) with \( a_{\lambda} \in \mathcal{A} \). Let \( H \) be \( \sum_{\lambda \in \Lambda} a_{\lambda} \lambda^{\mu} H(P, C_{\lambda}) \). Then decompose this \( H \) into four blocks:

\[
H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.
\]

Then if \( H_{22} \) is nonsingular, it is easy to see that \( H_{22}^{-1} H_{21} \) is a stabilizing controller of \( P \) (and \( H(P, H_{22}^{-1} H_{21}) = H \)). Thus \( P \) is stabilizable, which means that (i) holds.

For the case where \( H_{22} \) is singular, we can modify this \( H_{22} \) nonsingular. The reader should refer to [4].

\[\square\]

4 Local-Global Principle and Simultaneous Stabilizability

We consider in this section the simultaneous stabilization problem. Analogously to the previous section, we attempt to use the local-global principle.

The following is a local-global principle with a fixed stabilizing controller.

**Theorem 4.1** Let \( P_0 \) and \( P_1 \) be plants in \( \mathcal{P}^{m \times m} \). Let \( C \) denote a transfer matrix of \( \mathcal{F}^{m \times n} \).

Then the following statements are equivalent:
(i) The plants $P_0$ and $P_1$ are simultaneously stabilized by $C$.

(ii) There exists a finite subset $\Lambda$ of $A$ such that (a) $\sum_{\lambda \in \Lambda} \lambda = 1$ and (b) for each $\lambda \in \Lambda$, both $P_0$ and $P_1$ are simultaneously $A_\lambda$-stabilized by $C$.

**Proof.** Because “(i)$\Rightarrow$(ii)” is obvious, we show “(ii)$\Rightarrow$(i)” only.

For each $\lambda \in \Lambda$, $H(P_0, C)$ and $H(P_1, C)$ are over $A_\lambda$. Let $\mu$ be a sufficiently large positive integer such that for every $\lambda \in \Lambda$, $\lambda^\mu H(P_0, C)$ and $\lambda^\mu H(P_1, C)$ are over $A$. Then there exists $a_\lambda$’s in $A$ such that $\sum_{\lambda \in \Lambda} a_\lambda \lambda^\mu = 1$.

Obviously, $\sum_{\lambda \in \Lambda} a_\lambda \lambda^\mu H(P_0, C)$ and $\sum_{\lambda \in \Lambda} a_\lambda \lambda^\mu H(P_1, C)$ are over $A$ and are equal to $H(P_0, C)$ and $H(P_1, C)$, respectively. Thus $C$ is a stabilizing controller of both $P_0$ and $P_1$, which implies that $C$ is a simultaneously stabilizing controller of $P_0$ and $P_1$.

Now that we have obtained Theorem 4.1, we have another problem whether or not the simultaneous stabilizability over $A$ is equivalent to the simultaneous stabilizabilities over local rings of $A$. We give the result that they are not equivalent as follows.

**Theorem 4.2** Let $P_0$ and $P_1$ be plants in $\mathcal{P}^{n \times m}$. Consider the following statements:

(i) The plants $P_0$ and $P_1$ are simultaneously stabilizable.

(ii) There exists a finite subset $\Lambda$ of $A$ such that (a) $\sum_{\lambda \in \Lambda} \lambda = 1$ and (b) for each $\lambda \in \Lambda$, both $P_0$ and $P_1$ are simultaneously $A_\lambda$-stabilizable.

Then, (i) implies (ii). However, (ii) does not imply (i) in general.

This is a main result of this paper and is proved in the next section.

**5 Proof of Theorem 4.2**

In this section, we prove Theorem 4.2. Because “(i)$\Rightarrow$(ii)” is obvious, we will prove only that (ii) does not imply (i) in general. To prove this, it is sufficient to show an example that (ii) holds but (i) does not
hold. As an example, we employ Anantharam’s example [7]. He considered the case $\mathcal{A} = \mathbb{Z}[\sqrt{-5}] = \{u + v\sqrt{-5} \mid u, v \in \mathbb{Z}\}$, where $\mathbb{Z}$ denotes the set of integers (This ring [10, pp.134–135] is isomorphic to $\mathbb{Z}[x]/(x^2 + 5)$ and is an integral domain but not a unique factorization domain. In fact, $6 \in \mathbb{Z}[\sqrt{-5}]$ has two factorizations, $2 \cdot 3$ and $(1 + \sqrt{-5}) \cdot (1 - \sqrt{-5})$. We let $\mathcal{Z} = \{0\}$. Anantharam [7] showed that a plant $(1 + \sqrt{-5})/2$ does not admit a coprime factorization but is stabilized by $(1 - \sqrt{-5})/(-2)$. We use these transfer functions. Let $p_0 = (1 + \sqrt{-5})/2$ and $p_1 = (1 - \sqrt{-5})/(-2)$.

In the following, we first present the controller parametrizations of $p_0$ and $p_1$, next show that they are not simultaneously stabilizable and then show that they are simultaneously stabilizable over $\mathcal{A}_\lambda$ for every $\lambda \in \Lambda$.

**Controller Parametrization** Let $S(p)$ denote the set of all stabilizing controllers of $p$.

The controller parametrization of $p_0$ is given in [11], which is given as

$$S(p_0) = \left\{ \frac{2r_0 + (1 - \sqrt{-5})}{-(1 + \sqrt{-5})r_0 - 2} \mid r_0 \in \mathcal{A} \right\}. \quad (2)$$

In the following, we give the parametrization of all stabilizing controllers of $p_1$. Recall that this $p_1$ is originally a stabilizing controller of $p_0$. Thus, from the symmetry, $p_0$ is a stabilizing controller of $p_1$. Once we have found a stabilizing controller, we can obtain the parametrization of stabilizing controller of $p_1$ by the method of [5].

Let $\mathcal{H}(p_1)$ be the set of all $H(p_1, c)$’s with all stabilizing controllers $c$. Then, from [5], this $\mathcal{H}(p_1)$ is given as

$$\mathcal{H}(p_1) = \left\{ \begin{pmatrix} -3 & -1 + \sqrt{-5} \\ -1 - \sqrt{-5} & -2 \end{pmatrix} Q \begin{pmatrix} -2 & -1 + \sqrt{-5} \\ -1 - \sqrt{-5} & -3 \end{pmatrix} \begin{pmatrix} -2 & -1 + \sqrt{-5} \\ -1 - \sqrt{-5} & -3 \end{pmatrix} + \begin{pmatrix} -2 & -1 + \sqrt{-5} \\ -1 - \sqrt{-5} & -2 \end{pmatrix} \right\} \quad (3)$$

Q \in \mathbb{A}_2 \times 2$.
Use $q_{ij}$ to denote the $(i, j)$-entry of the matrix $Q \ (i, j = 1, 2)$. Then $\mathcal{H}(p_1)$ reads as

$$
\mathcal{H}(p_1) = \left\{ \begin{bmatrix} (1 - \sqrt{-5})r_1 & -(2 + \sqrt{-5})r_1 \\ 2r_1 & (1 - \sqrt{-5})r_1 \end{bmatrix} + \begin{bmatrix} -2 & -1 + \sqrt{-5} \\ -1 - \sqrt{-5} & -2 \end{bmatrix} \mid q_{ij} \in \mathcal{A} \ (i, j = 1, 2) \right\}
$$

(4)

where

$$
r_1 = (1 + \sqrt{-5})q_{11} + 2q_{21} - (2 - \sqrt{-5})q_{12} + (1 + \sqrt{-5})q_{22}.
$$

(5)

Because any stabilizing controller can be obtained from the $(2,1)$- and $(2,2)$-entries of the matrix in (3), we are interested in the set of $r_1$’s of (5), that is, the ideal generated by $(1 + \sqrt{-5})$, $2$, and $-(2 - \sqrt{-5})$.

By letting $q_{11} = 1$, $q_{12} = -1$, $q_{21} = -1$, $q_{22} = 0$, we see that $r_1$ is equal to 1. This implies that the set of $r_1$’s of (5) is identical to $\mathcal{A}$. Hence the parameterization of stabilizing controllers of $p$ is given as

$$
\mathcal{S}(p_1) = \left\{ \frac{2r_1 - 1 - \sqrt{-5}}{(1 - \sqrt{-5})r_1 - 2} \mid r_1 \in \mathcal{A} \right\}.
$$

(6)

The denominator of (6) cannot be zero because in such case, $r = \frac{1}{2}(1 - \sqrt{-5})$, which is not in $\mathcal{A}$.

**Non-Simultaneous-Stabilizability** Suppose that there exists a simultaneously stabilizing controller of $p_0$ and $p_1$. Then, from (2) and (6), there exist $r_0$ and $r_1$ in $\mathcal{A}$ such that

$$
\frac{2r_0 + (1 - \sqrt{-5})}{-(1 + \sqrt{-5})r_0 - 2} = \frac{2r_1 - 1 - \sqrt{-5}}{(1 - \sqrt{-5})r_1 - 2}
$$

(7)

holds. Let $a_0$, $b_0$, $a_1$, $b_1$ be integers with $r_0 = a_0 + \sqrt{-5}b_0$ and $r_1 = a_1 + \sqrt{-5}b_1$. Then (7) can be decomposed into the real part and the imaginary part equations as follows:

$$
\begin{cases}
-4 + 4a_0a_1 + 10b_0 + 10b_1 - 20b_0b_1 = 0, \\
-2a_0 - 2a_1 + 4a_1b_0 + 4a_0b_1 = 0.
\end{cases}
$$

(8)
By solving (8), $b_0$ can be expressed by $a_1$ and $b_1$ as follows:

$$b_0 = \frac{2 + 2a_1^2 - 9b_1 + 10b_1^2}{5 + 4a_1^2 - 20b_1 + 20b_1^2}. \quad (9)$$

Recall now that the variables $b_0, a_1, b_1$ have some integer values. Even so, we here show that the right hand side of (9) cannot be integer. Now decompose each of the numerator and the denominator of (9) into two parts within parentheses:

$$\frac{(1 + 2a_1^2) + (1 - 9b_1 + 10b_1^2)}{(2 + 4a_1^2) + (3 - 20b_1 + 20b_1^2)}.$$

Then we have $0 < 1 + 2a_1^2 < 2 + 4a_1^2$ for every $a_1 \in \mathbb{Z}$ also $0 < 1 - 9b_1 + 10b_1^2 < 3 - 20b_1 + 20b_1^2$ for every $b_1 \in \mathbb{Z}$. Thus the right hand side of (9) is greater than zero and less than one for any $a_1, b_1 \in \mathbb{Z}$. This implies that $b_0$ cannot be an integer and that the equation (7) of $r_0$ and $r_1$ does not have a solution.

Hence we conclude that the plants $p_0$ and $p_1$ are not simultaneously stabilizable, that is, $p_0$ and $p_1$ do not satisfy (i) of Theorem 4.2.

**Simultaneous Stabilizability over $A_\lambda$ for $\lambda \in \Lambda$**

First, we let $\Lambda = \{-2, 3\}$. Then $\sum_{\lambda \in \Lambda} \lambda = 1$.

Because the denominators of $p_0$ and $p_1$ are 2 and $-2$, respectively, both $p_0$ and $p_1$ are in $A_{-2}$. Thus $p_0$ and $p_1$ are $A_2$-stabilized by zero, which is a simultaneously $A_{-2}$-stabilizing controller.

Consider $A_3$. Observe that $p_0$ can be rewrite as $3/(1 - \sqrt{-3})$ and $p_1$ as $-3/(1 + \sqrt{-3})$. Thus $(1, p_0^{-1})$ and $(1, p_1^{-1})$ are coprime factorizations of $p_1$ over $A_3$.

Let $S(p)$ denote the set of all $A_\lambda$-stabilizing controllers of $p$. Then we have, by Youla-parametrization,

$$S(p_0)_3 = \{(1 + p_0^{-1}r_0)/r_0 \mid r_0 \in A_3 \setminus \{0\}\},$$

$$S(p_1)_3 = \{(1 + p_1^{-1}r_1)/r_1 \mid r_1 \in A_3 \setminus \{0\}\}.$$
equation

\[(1 + p^{-1}_0 r_0) r_1 = (1 + p^{-1}_1 r_1) r_0\]

over \(A_3\). By the straightforward calculation, we see that \((r_0, r_1) = (-1, -3)\) is one of solutions. Its simultaneously \(A_3\)-stabilizing controller is \(c = (-1 + \sqrt{-5})/(1 + \sqrt{-5})\). Thus, \(p_0\) and \(p_1\) are simultaneously \(A_3\)-stabilizable.

Thus these \(p_0\) and \(p_1\) satisfy (ii) of Theorem 4.2. Therefore, (ii) of Theorem 4.2 does not imply (i) of Theorem 4.2 in general.

6 Conclusions

In this paper, we have addressed the simultaneous stabilization problem. Primary goal of the current study is to obtain some criterion of solving the simultaneous stabilization problem without coprime factorizability. This is not given in this paper yet. Even so we have shown that the simultaneous stabilizability over the set of stable causal transfer functions cannot be given by the simultaneous stabilizabilities over local rings in general. On the other hand, when a plant admits a doubly coprime factorization, we can consider that the local-global principle become trivial in the sense that \(\Lambda = \{1\}\). Based on these, we need further to investigate the criterion that the local-global principle can be applied to the simultaneous stabilization problem.

References


