Abstract: This paper is concerned with the design of an adaptive servo controller for tracking variable references in a distributed collector solar field. The structure proposed is made up of three main blocks: A motion planner, an incremental controller and an adaptation mechanism. The motion planner selects the time profile of the manipulated variable (oil flow) such that the plant state (oil temperature distribution along the solar field) is driven between successive equilibrium states as specified. This is done on the basis of a simplified, yet distributed parameter, model and uses the methods of flat systems and the concept of orbital flatness. In order to stabilize the actual oil temperature around this nominal path, a linear controller is used. This control law is then modified according to a Lyapunov function strategy for incorporating adaptation through the adjustment of a parameter which conveys the most significative plant uncertainty. The approach is illustrated through simulations performed in a detailed solar field model.

Keywords: Flat Systems, Orbital flatness, Motion Planning, Distributed Systems, Solar Energy, Servo problems, Adaptive.
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The paper is organized as follows: After this introduction, the distributed collector solar field considered is briefly described in section 2. The problem of motion planning for the field is solved in section 3 where the flatness based motion planning is briefly reviewed and it is shown that the plant considered is orbitally flat. In section 4 the incremental regulator which stabilizes the plant state around the nominal trajectory is designed. Adaptation is introduced in section 5 and section 6 draws conclusions.

2. DISTRIBUTED COLLECTOR SOLAR FIELDS.

Distributed collector solar fields aim at collecting energy from direct sun radiation and storing it in the thermal form. Fig. 1 shows an overall view of the ACUREX field of Plataforma Solar de Almeria, located in the south of Spain, which is the subject of this paper. This plant is made of curved mirrors which concentrate direct incident sun light in a pipe located at their focus. Inside this pipe flows an oil able to store thermal energy. In very general terms, the control objective consists in manipulating the oil flow such that the temperature of the oil has some prescribed value. Although many different strategies for achieving this objective have been considered in the rich literature devoted to the subject (see Silva et al., 2003) for an up to date review as well as a more detailed description of the plant), this is mainly concerned with the regulation problem and disturbance rejection. Opposite, in this paper the problem of reference tracking is considered.

3. FLATNESS BASED MOTION PLANNING.

Flatness was first defined in (Fliess et al., 1992; Fliess et al., 1995) using the formalism of differential algebra (which justifies the qualifier "flat"). In simple terms, a system is said to be flat if it is possible to find a set of outputs, called the flat outputs, equal in number to the number of inputs and such that all states and inputs can be determined from these outputs without integration. If the system variables are to follow some given time profile, this is specified in terms of the flat outputs, and the above functions are then used to compute the corresponding inputs. This is, in rough terms, the motion planning problem (Lynch and Rudolph, 2002). For SISO systems, the planning performed in this way is equivalent to have an "artificial" output through which it is possible to perform exact feedback linearization (Martin et al., 2001).

Fig. 2 shows the block diagram of a general motion planning based servo control structure. The dominant dynamics of the field is described by the simple hyperbolic partial differential equation (PDE):

$$\frac{\partial T(z,t)}{\partial t} + u(t) \frac{\partial T(z,t)}{\partial z} = \alpha R(t)$$  \hspace{1cm} (1)

where $T(z,t)$ denotes the oil temperature at position $z$ and at time $t$, $u$ is the oil velocity (proportional to flow), taken as the manipulated variable, $R$ is a known function of solar radiation and $\alpha$ is a parameter which is assumed to be constant and known. The length of the pipe is denoted by $L$. The state of this distributed parameter system is described at each time $t$ by the function \{ $T(z,t), 0 \leq z \leq L$\}.

Actually, eq. (1) is not an accurate model of the solar field. Nevertheless it reflects the dominant model of the real plant well enough to be used for designing a motion planner. Simulations of the controlled are thus made with a much more detailed model which takes into account the nonlinear thermodynamic characteristics of the oil.
Although for (1) there is probably not a flat output, it is possible to introduce a time scaling such that the system becomes flat (Fliess et al., 1995; Guay, 1999; Respondek, 1998; Vollmer and Raisch, 2003). Such a system is then said to be orbitally flat. Thus, the solution to the problem at hand is obtained by introducing a change of variable $\tilde{\tau}(t)$ such that, in the new coordinate space $(z, \tilde{\tau})$ a flat output $\tilde{y}$ is found

$$\tilde{y}(\tilde{\tau}) = h(y(z, \tilde{\tau}), u, \dot{u}, \ldots)$$

(2)

where $h$ is a function to be found.

Once the flat output is found, it is possible to describe all trajectories $y(z, \tau), u(\tau)$ satisfying the transformed PDE as a function of the flat output and its derivatives, in particular

$$y(z, \tau) = \varphi(\tilde{y}, \dot{\tilde{y}}, \ddot{\tilde{y}}, \ldots)$$

(3)

$$u(\tau) = \chi(\tilde{y}, \dot{\tilde{y}}, \ddot{\tilde{y}}, \ldots)$$

(4)

Eqs. (3,4) provide the solution to the motion planning problem. Indeed, once a given shape for $\tilde{y}$ is imposed, (4) yields the input profile which drives the state $y(z, \tau)$ as required. Motion planning for the ACUREX solar collector field is briefly explained hereafter. Details are contained in (Igreja et al., 2004).

3.1 Orbital flatness

According to the approach of orbital flatness, consider the change of time scale

$$\tau(t) = \int_0^t u(\sigma) d\sigma$$

$$\frac{d\tau}{dt} = u(t)$$

(5)

This change of variable introduces a "natural" time scale associated to oil flow which, as shown in (Silva et al., 2003), linearizes the plant model, forcing the characteristic lines of (1) to become straight lines. Assumptions should be made (Vollmer and Raisch, 2003) that the mapping between $t$ and $\tau$ is bijective, that $\tau$ is a monotonically function of $t$ and that it goes to infinity if and only if $t$ goes to infinity. The validity of these assumptions is ensured by natural physical constraints in the practical problem at hand. Under these hypothesis, "real" time $t$ can be recovered from the transformed time $\tau$ from

$$t(\tau) = \int_0^\tau \frac{1}{u(\sigma)} d\sigma$$

(6)

In the time scale $\tau$ eq. (1) becomes

$$\frac{\partial T(z, \tau)}{\partial \tau} + \frac{\partial T(z, \tau)}{\partial z} = f(\tau)$$

(7)

where

$$f(\tau) \triangleq \frac{\alpha R(t(\tau))}{u(t(\tau))}$$

(8)

is the new manipulated variable.

In (Igreja et al., 2004) it is shown that the gradient of the outlet oil temperature, given by

$$\left.\frac{\partial T(z, \tau)}{\partial z}\right|_{z=L} = y(\tau)$$

(9)

is a flat output. Furthermore, from this variable, the manipulated variable and the plant state can be expressed in transformed time $\tau$ as

$$f(\tau) = \sum_{k=0}^{\infty} \frac{L^k}{k!} y^{(k)}(\tau)$$

(10)

$$T(z, \tau) = T_0 + \sum_{k=1}^{\infty} \frac{L^k}{k!} y^{(k-1)}(\tau) - \sum_{k=1}^{\infty} \frac{(L-z)^k}{k!} y^{(k-1)}(\tau)$$

(11)

Here, $T_0 \equiv T(0, \tau)$ is the inlet oil temperature, assumed constant. The above expressions provide the algebraic expressions needed for dynamic motion planning. The trajectories depend only on the knowledge of the inlet temperature and on the successive derivatives with respect to time of the flat output. Planning is possible as long as the series converge.

3.2 Trajectory generation

Motion planning connects stationary states, for which

$$\frac{dT_{ss}(z)}{dz} = f_{ss}$$

(12)

and hence

$$T_{ss}(z) = f_{ss}z + T_0$$

(13)

where $f_{ss}$ is the gradient of the temperature with respect to space and $T_{ss}$ is the temperature along the pipe in steady state. Planning is made (fig. 1) such that the temperatures along the pipe move from the stationary state

$$T(z, 0) = C_1 z + C_{01}$$

(14)

with

$$T_0 = C_{01}; \quad T(L, 0) = C_1 L + C_{01}$$

(15)

to the new stationary state

$$T(z, 0) = C_2 z + C_{01}$$

(16)

with

$$T(L, \tau^*) = C_2 L + C_{01}$$

(17)

Hence

$$C_1 = \frac{T(L, 0) - T_0}{L}$$

(18)

$$C_2 = \frac{T(L, \tau^*) - T_0}{L}$$

(19)

The trajectory connecting two stationary states at times $\tau = 0$ and $\tau = \tau^*$ is defined by an exponential type Gevrey function of class $\alpha$ (Rudolph et al., 2003), $\Phi_{y}\sigma$. This function (fig. 3) provides a profile for changing the flat output, given by:

$$y(\tau) = C_1 + (C_2 - C_1) \Phi_{y}\sigma \left(\frac{\tau}{\tau^*}\right)$$

(20)

in which

$$\Phi_{y}\sigma(0) = 0 \quad \tau \leq 0$$

$$\Phi_{y}\sigma(1) = 1 \quad \tau \geq \tau^*$$

(21)

(22)
and all the derivatives computed at 0 and $\tau^*$ being zero. This allows a smooth transition between stationary states. The process can be repeated for an arbitrary number of successive transitions between stationary states.

Figs. 3-7 illustrate the trajectories generated for a planning with $L = 180$, $\alpha R(t) = 1$ and $T_0 = 0$, and in which 3 successive transitions take place. Initially, the planning is performed in transformed time $\tau$. Fig. 3 shows the temperature evolution in transformed time $T(z, \tau)$, and fig. 4 shows the temperature gradient, $y(\tau)$, and the corresponding input, $f(\tau)$, in transformed time. Once the planning is complete in transformed time, real time is recovered using eq. (6). This is shown in fig. 5. Then the planning is referred to real time by using the relation expressed in fig. 5 to transform the time scale of figs. 3 and 4 to get figs. 6 and 7.

4. SERVO CONTROL

The problem of tracking a reference trajectory for flat SISO systems can be formulated as follows (Henson and Seborg, 1997; Martin et al., 2001): Given a nonlinear system of finite dimension $n$ and state $x$ described by

$$\dot{x} = F(x, u)$$

(23)

with flat output

$$\dot{y} = h(x)$$

(24)

find a controller able to track any reference trajectory $t \rightarrow (x_r(t), u_r(t))$.

Since the dynamics admits a flat output it is possible, by a change of variable, to transform the system to the normal form
Planer T (t) u(t) trajectories for oil flow (or velocity) and for outlet oil to use the above solution for the problem at hand. Under these circumstances it does not seem possible that temperature is only measured at the output. (orbital) flat output and the technological limitation that the output to control is not coincident with the infinite dimension when described by ODE’s), the fact of distributed parameter type (meaning that it is of $\tau$ in normal form in the domain of transformed time that it is only possible to write the system equations in the special case of the ACUREX distributed collector solar field. 

By making

$$\nu = \frac{1}{n+1} \Delta y$$

the closed loop error satisfies

$$\Delta y = -K \Delta y$$

where

$$\Delta y = y(t) - y_r(t)$$

Tracking is therefore reduced to closed loop control of a series of $n$ integrators whose error dynamics is linear and given by (27). Fig. 2 shows a general block diagram for this servo controller. Selecting the gains of the vector $K$ in a suitable way it is possible to impose a stable dynamics for the closed loop error (Martin et al., 2001).

In the special case of the ACUREX distributed collector solar field the main difficulty consists in the fact that it is only possible to write the system equations in normal form in the domain of transformed time. Other difficulties are the fact that the system is of distributed parameter type (meaning that it is of infinite dimension when described by ODE’s), the fact that the output to control is not coincident with the (orbital) flat output and the technological limitation that temperature is only measured at the output.

Under these circumstances it does not seem possible to use the above solution for the problem at hand. However, as shown above, it is possible to obtain trajectories for oil flow (or velocity) and for outlet oil temperature, denoted respectively $u_r(t)$ and $T_{or}(t)$. Having these signals it was then decided to close the loop through a PID controller which produces a fast compensation of the deviations with respect to the reference trajectory ($T_{or}(t), u_r(t)$). These deviations are due to disturbances and modelling errors. Fig. 8 shows the servo control based on motion planning proposed for the ACUREX field.

The PID control law is given by

$$u(t) = u_r(t) + \Delta u(t)$$

5. ADAPTATION

The parameter $\alpha$ in eq. 1 reflects not only the mirror efficiency but is also influenced by the nonlinear dependence of oil specific heat on temperature. Changing the operating point (as in the servo problem)
where \( \tilde{f} \) is the planned input, assuming that \( \alpha R_{\text{nom}}(t) = 1 \). Furthermore, the estimate \( \hat{\alpha} \) of \( \alpha \) is updated according to

\[
\hat{\alpha}(t) = -\gamma \int_{0}^{t} R(\zeta)e(\zeta)d\zeta + \hat{\alpha}(0)
\]

In this way a PI control law is recovered, incorporating at the same time an estimate of \( \alpha \).

For justifying the above adaptive control law, an argument based on the approximating (1) by a lumped parameter model is presented. For that sake, consider \( N \) points along the pipe, equally distant in space at positions \( jh, \ h = L/N \ j = 1, \ldots, N \) and let \( T_j \) be the temperature at the corresponding point. Then (Barão et al., 2002), eq. (1) can be approximated by a set of ODE’s of which the last one is

\[
\dot{T}_N = -\frac{u(t)}{h}(T_N - T_{N-1}) + (\hat{\alpha} - \tilde{\alpha})R
\]

where

\[
\tilde{\alpha} = \hat{\alpha} - \alpha
\]

Inserting the value of \( u \) given by (31) yields

\[
\dot{T}_N = -\left[\frac{\hat{\alpha}(t)R(t)}{f(t)} - \frac{K_C}{f(t)}e(t)\right]T_N - T_{N-1} + (\hat{\alpha} - \tilde{\alpha})R
\]

Since the planned input verifies

\[
\frac{T_N - T_{N-1}}{h} \approx \tilde{f}
\]

it finally comes that the outlet oil temperature verifies the ODE

\[
\dot{T}_N = K_C e(t) + \hat{\alpha} R
\]

From this equation, by repeating the arguments in (Barão et al., 2002) based on a joint Lyapunov function for estimation and control, it is then possible to obtain the adaptation rule, while proving that the overall system is stable and that the tracking error will tend to zero.

Figs. 13-17 illustrate the procedure. Fig. 17 shows the quotient

\[
\frac{T_N - T_{N-1}}{hf}
\]

After the initial adaptation transient, it becomes close to 1, thereby verifying the assumption underlying (36).

6. CONCLUSIONS

This paper has shown how to develop a tracking adaptive controller for a distributed collector solar field, using a motion planner based on orbital flatness. The procedure followed can be applied to other similar plants, such as moisture control.
Fig. 13. Adaptive closed loop, $K_C = 0.1$, $\gamma = 1 \times 10^{-8}$.

Fig. 14. Planned oil flow and oil flow yielded by the closed loop with adaptation [$m^3/s$].

Fig. 15. Adaptive closed loop with $K_C = 0.1$, $\gamma = 1 \times 10^{-8}$: Plant outlet oil temperature and reference to track.

7. REFERENCES


