Vehicle Control Augmentation
Based on an Integrated Design Methodology

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Abstract

This paper proposes a new design strategy of vehicle control augmentation system. Motivated by an integrated design methodology for mechanical control system, the paper brings into focus the phase property of a vehicle dynamics in order to improve tracking performance for the reference input by a pilot. The proposed design criterion turns out to be in good agreement with Neal-Smith criterion. It is shown that the control augmentation system design with this criterion is achieved by means of a $\gamma$-positive real control technique.

1 Introduction

For easy and precise piloting, some handling quality criteria for vehicles such as cars or aircrafts are proposed. Vehicles are desired to have suitable dynamical properties which meet handling quality criteria. However, it is often difficult to optimize the structure of vehicles for improvement of dynamical properties, because requirements for low costs or other practical restrictions usually have priority over handling quality. Therefore, additional control systems aiming at improvement of vehicle dynamics are widely implemented nowadays. Examples are power steering, traction control system, pitch/yaw damper, and more complicated feedback control systems. Let us generically call these control systems CAS (control augmentation system).

There are two popular approaches for CAS design. If a vehicle model which has an ideal dynamical property is given in advance, model matching/following are useful approach. However, it is not easy to determine the ideal vehicle model in many cases. If a pilot dynamics is modeled as LTI system, we can design CAS by ordinary feedback control technique. However, individual variation of the pilot makes such modeling difficult. Therefore, it would be meaningful to discuss CAS design problem from another new system theoretical point of view.
As is shown in later, a problem setting of CAS design is similar to that of integrated design, which is well-known in the field of mechanical control system design. This suggests that our problem is a class of BMI problem and difficult to obtain a globally optimal solution with a reasonable computing cost in general. However, Iwasaki [1] proposed a tractable integrated design strategy by separation. The separation strategy means that first a plant is designed to have the desirable open-loop property for optimal closed-loop performance, then a controller is designed to achieve optimal closed-loop performance. From numerical analysis and theoretical results, Iwasaki concluded that we should design the plant so that maximum phase lag does not exceed 180(deg.) within the desired control bandwidth, for a servo control of mechanical control system. Therefore, from the similarity of both problem settings, it is natural to expect that the similar separation strategy will have effectiveness to CAS design problem as well.

This paper proposes a new design strategy for CAS design by a similar methodology to that used in structure/control integrated design of mechanical control systems. Following the presented strategy, this paper bring into focus the relationship between phase crossover frequency of a vehicle and optimal $H_2$ tracking performance. Then, from the results of $H_2$ performance analysis, the proposed design criterion on phase crossover bandwidth turns out to be in good agreement with Neal-Smith criterion, which is a well-known criterion for aircraft handling quality evaluation. Finally, CAS design with the presented criterion is achieved by means of the $\gamma$-positive real control technique.

Let us make clear the distinction of two types of crossover frequency which is often used in this paper. For a transfer function $H(s)$:

**Phase crossover frequency:** The smallest $\omega_p > 0$ such that $\arg(H(j\omega_p)) = -\pi$.

**Positive real crossover frequency:** The smallest $\omega_r > 0$ such that $\arg(H(j\omega_r)) = -\pi/2$.

## 2 Vehicle control augmentation system design

A typical example of CAS design is an aircraft pitch tracking control (Fig. 1). For the vehicle, the airframe is usually designed in advance, so a designer can only design the assistant controller (CAS in Fig. 1). The vehicle is feedback-controlled by the pilot. An experienced (or “optimized”) pilot can achieve optimal performance on quick and accurate tracking the pitch angle $\theta$ to the reference input $\theta_c$. It is the main and difficult problem how to design CAS to improve the optimal tracking performance without causing instability.

![Figure 1: Aircraft pitch tracking control.](image-url)
3 Problem setting

Then, based on the above knowledge, let us define a CAS design problem from a system theoretical point of view. Consider the generalized plant $P_K$:

$$
\begin{bmatrix}
g \\
y
\end{bmatrix} = G \begin{bmatrix}
f \\
u
\end{bmatrix}, \quad f = \Sigma_K g
$$

(1)

where $u, y$ are control input/output, and $f, g$ are general input/output. Note that $G$ is given and $\Sigma_K$ is a subsystem which we can design. Then, consider the feedback system in Fig. 2 for the $P_K$, where $\Sigma_H$ is another controller which we can also design. Assume that $\Sigma_K, G$

![Diagram](image)

Figure 2: Control augmentation system design.

and $\Sigma_H$ are finite dimensional LTI systems.

Note that the subsystems $\Sigma_K$, $G$, and $\Sigma_H$ correspond to CAS, the airframe and the pilot respectively. Therefore, the CAS design problem is to design $\Sigma_K$ and $\Sigma_H$ for given $G$. Furthermore, in general setting of CAS design, available power of $u$ is more limited than that of $f$. In what follows, we will restrict our interest to CAS design problem for optimization of $H_2$ tracking performance.

By using the above framework, let us define the CAS design problem as follows.

**Problem 1 (Control augmentation system design problem)** Let $G$ and $\gamma_u > 0$ be given. Find a pair of $\Sigma_H$ and $\Sigma_K$ which minimize the optimal $H_2$ tracking performance

$$
\gamma_e(\gamma_u) := \min \|T_e\|_2 \quad \text{such that} \quad \|T_u\|_2 < \gamma_u,
$$

(2)

where we define

$$
\begin{bmatrix}
e \\
u
\end{bmatrix} = \begin{bmatrix}
T_e \\
T_u
\end{bmatrix} r.
$$

(3)

The index $\gamma_u$ corresponds the limit of control power (e.g. average power of the control stick force). If there exist $\Sigma_H$ and $\Sigma_K$ which optimize the optimal tracking performance for the given $\gamma_u$, the $\Sigma_K$ is an optimal CAS.

If $\Sigma_K$ is restricted to passive component (i.e. $\Sigma_K = \text{block-diag}[p_1I, \ldots, p_rI]$ with design parameters $p_1, \ldots, p_r$), Problem 1 is reduced to an integrated design problem of structure
and control system. This means that Problem 1 is a class of BMI problem and difficult to obtain a globally optimal soliton with a reasonable computing cost in general.

However, Iwasaki [1] proposed a tractable design strategy for the integrated design version of Problem 1, by separation. The separation strategy means that first $\Sigma_K$ is designed to make $P_K$ meet the open-loop property which is desire to achieve optimal closed-loop performance, then $\Sigma_H$ is designed to achieve optimal closed-loop performance. Iwasaki considered the $H_2$ tracking performance for unit step reference input $r$, and concluded that the phase crossover frequency of $P_K$ is desired to be higher than control bandwidth.

## 4 Bandwidth analysis and the optimal closed-loop performance

From the similarity between the problem setting of CAS design and integrated design, it is natural to expect that the similar separation strategy will have effectiveness to our problem. Then, let us make analysis how the phase crossover frequency of vehicle is related to closed-loop tracking performance limit which is achievable by pilot. Consider a pitch tracking control design problem (Fig. 1) in this section.

Iwasaki solved $H_2$ optimization problem in (2) for given $G$, $\gamma_u$ and many fixed $\Sigma_k$'s, and observed that closed-loop performance $\gamma_e$ is optimized for some set of $P_K$. Therefore, let us solve the similar problem for aircraft models with 52 variations of CAS, which are from the flight investigation by Neal and Smith [2]. In Fig. 3, achieved closed-loop $H_2$ performance

Figure 3: Adequate phase crossover bandwidth for optimal $H_2$ performance.

$\gamma_e$ and open-loop phase crossover frequency $\omega_p$ for each aircraft configuration are plotted. We can observe that $H_2$ performance is optimized for the configurations which has about
\( \omega_p \geq 8.7 \text{ (rad/s)} \) (marked by ‘o’). In the above meaning, let us call them “good” configuration.

Neal and Smith [2] evaluated the closed-loop performance by numerical index “Pilot Rating” (PR), which is systematically determined from pilot comments and primary determined by how quickly and precisely the pilot could control pitch angle. The range of PR is from 1 (best) to 10 (worst). In Fig. 4, PR and open-loop phase crossover frequency \( \omega_p \) for each aircraft configuration are plotted, where PR for each configuration is from Neal and Smith’s work [2]. We can observe that PR is also optimized for “good” configurations. The average PR for “good” configurations is 3.95.

Then, let us show the desired positive real bandwidth in Neal-Smith criterion naturally follows from our \( H_2 \) tracking performance analysis. Neal and Smith proposed a criterion for aircraft CAS design what is called “Neal-Smith criterion”. One of the points of their criterion is that closed-loop positive real crossover frequency \( \omega_{rp} \) is desired to be higher than \( 3.0 \sim 3.5 \text{ (rad/s)} \) for good tracking performance. In Fig. 5, we can observe that “good” configurations in view of open-loop phase crossover bandwidth results \( \omega_{pr} \geq 3.5 \text{ (rad/s)} \) without one exception.

From the above bandwidth analysis, we can observe that

1. If open-loop phase crossover bandwidth is sufficiently large, then both \( H_2 \) tracking performance and PR are optimized.

2. For this vehicle, about \( \omega_p = 8.7 \text{ (rad/s)} \) is enough for optimal \( H_2 \) tracking performance and PR.

3. The result of \( H_2 \) analysis shows good agreement with Neal-Smith criterion, for “good” configurations in view of open-loop phase crossover bandwidth.

![Figure 4: Adequate phase crossover bandwidth for optimal Pilot Rating.](image-url)
These observations suggest that the separation strategy has effectiveness for our CAS design problem as similar for mechanical control system design. From the observation 2, it seems that the desired control bandwidth for optimal PR is about 8.7 (rad/s) for this vehicle. This gives a new design criterion for CAS design from integrated design point of view.

5 CAS design via $\gamma$-positive real control

Let us show a numerical example of optimal CAS design by the separation strategy. In this strategy, improvement of phase crossover bandwidth is important. The airframe dynamics

\[
G(s) := \frac{1}{\tau_a s + 1} \times \frac{K_{\theta}(\tau_{g2}s + 1)}{s \left( \frac{1}{\omega_{xp}}s^2 + 2\frac{\zeta_{xp}}{\omega_{xp}}s + 1 \right)}.
\]  

(4)

$G$ is from Neal and Smith’s investigation [2] (configuration ‘1F’), where elevator actuator is modeled as a first order system ($\tau_a = 0.5$).

![Figure 5: Positive real bandwidth analysis in view of Neal-Smith criterion.](image)

![Figure 6: CAS design problem.](image)
Plant parameters are $K_\theta = 1$, $\tau_{\theta 2} = 1/1.25$, $\omega_{sp} = 2.2$, $\zeta_{sp} = 0.69$ respectively.

Following the discussion about the relationship between PR and phase crossover frequency in Section 4, a design criterion for optimal $\Sigma_K$ with respect to PR is given as follows;

**Criterion 1** *Phase crossover frequency of $P_K$ is more than 8.7(rad/s).*

We can easily design a $\Sigma_K$ which satisfies Criterion 1 by $\gamma$-positive real control technique[3]. Let us choose a transfer function $W(s)$ which satisfies Criterion 1;

$$W(s) := \frac{2}{0.05s^3 + 0.9s^2 + 4s + 2}. \quad (5)$$

We obtain a suitable $\Sigma_K$ by solving the following $H_\infty$ problem for given $0 < \gamma < 1$, which is equivalently obtained from $\gamma$-positive real control problem via Cayley transformation.

Find $\Sigma_K$ s.t. $\|G_\infty\|_\infty < \gamma$, \quad (6)

where $G_\infty := (G_\gamma - I)(G_\gamma + I)^{-1}$, $G_\gamma := W^{-1}P_K$, $P_K := (I + G\Sigma_K)^{-1}G\Sigma_K$.

Then, it is known that the following closed-loop phase property holds for $\gamma$-positive real system $G_\gamma$ in SISO case. This implies maximum phase difference between $W(s)$ and $P_K(s)$ is less than or equal to $\theta_\gamma$.

$$\forall \omega \in \mathbb{R}, \quad |\arg(G_\gamma(j\omega))| \leq \theta_\gamma, \quad \theta_\gamma := \tan^{-1}\left(\frac{2\gamma}{1 - \gamma^2}\right). \quad (7)$$

By solving the $H_\infty$ problem (7) for $\gamma = 9.882 \times 10^{-5}$, a suitable $\Sigma_K$ is obtained as follows;

$$\Sigma_K(s) = \frac{5.165s^4 + 26.01s^3 + 56.36s^2 + 50s + 8.283 \times 10^{-5}}{s^4 + 19.25s^3 + 102.5s^2 + 100s + 1.451 \times 10^{-4}}. \quad (8)$$

We can clearly see the improvement of phase crossover bandwidth in Fig. 7. The broken line in Fig. 7 is a Bode plot of $G(s)$. Phase crossover frequency of $G(s)$ is about 2.56(rad/s), therefore the vehicle does not satisfy our criterion. The solid line in Fig. 7 is a Bode plot of the closed-loop system $P_K(s)$ which is designed via $\gamma$-positive real control technique, where dash-dot line with marker ‘x’ is a Bode plot of weighting function $W(s)$ in (5). We can see that phase crossover frequency is improved to about 8.88(rad/s), therefore the closed-loop system $P_K(s)$ satisfies Criterion 1.

6 Conclusion

This paper proposes a new design strategy for vehicle CAS (control augmentation system) by a similar methodology to that used in structure/control integrated design. From $H_2$ performance analysis with Neal and Smith’s flight investigation data, it is observed that phase crossover bandwidth plays important role to improve the pilot rating, which principally depends on aircraft pitch tracking performance. This suggests that the separation strategy and the phase crossover bandwidth analysis have effectiveness for optimal design of not only mechanical control system but also vehicle CAS. An optimal CAS design criterion is given by the bandwidth analysis, and the feedback system which satisfies the criterion is designed by means of $\gamma$-positive real control.
Figure 7: Design result.

References

