Math 10B.
Practice Midterm Exam 1
October 22, 2018

Exam rules:

1. No electronic devices of any kind are allowed during this exam.

2. You may use one 2-sided US Letter sized page of notes, but no books or other assistance are allowed during this exam.

3. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Instructions:

1. Write your first and last name at the top of every page of this exam.

2. Write your solutions clearly and legibly. No credit will be given for illegible solutions.

3. Show the work leading to your solutions. Partial credit can only be given based on the work you show.

4. Please put a box around your final answer to each part.

→ If any question is not clear, ask for clarification.
1. A bumblebee is flying straight down with velocity 21 m/s when it accelerates upward at a constant 7 m/s².

(a) When is the bee's velocity 0?

\[
\text{constant acceleration of } +7 \text{ m/s}^2
\]

\[
\Rightarrow s''(t) = v'(t) = 7
\]

\[
\text{initial velocity is } -21 \text{ m/s}^2
\]

\[
\Rightarrow v(0) = -21
\]

So \[ v(t) = 7t + C \]

\[
\Rightarrow v(t) = 7t - 21
\]

Then \[ v(t) = 0 \iff t = 3 \]

(b) If the bee is 84 meters above the ground when it accelerates (that is, at time \( t = 0 \)), find its position function \( s(t) \) in meters above the ground.

From (a) we know \[ s'(t) = 7t - 21 \]

and we're given that \( s(0) = 84 \).

So \[ s(t) = \frac{7t^2}{2} - 21t + C \]

\[
\Rightarrow \int s(t) = \underbrace{\frac{7t^2}{2} - 21t + 84}_{s(t)}
\]
(c) Find the total distance traveled by the bee from time $t = 0$ to $t = 5$ seconds. That is, where $v(t)$ is the bee’s velocity, find:

$$\int_0^5 |v(t)|\,dt$$

See that $v(t) = s'(t) = 7t - 2$

and then $v(t)$ changes signs at $t = 3$.

So now:

$$\int_0^5 |v(t)|\,dt = \int_0^3 -v(t)\,dt + \int_3^5 v(t)\,dt$$

$$= \left[ s(3) - s(0) \right] + \left[ s(5) - s(3) \right]$$

$$= \left[ \frac{7}{2}(9) - 2(3) - 0 \right] + \left[ \frac{7}{2}(25) - 2(5) \right]$$

$$= \frac{7}{2}(9) + \frac{7}{2}(25) - \frac{7}{2}(9) + 2(3) - 2(5) + 2(3)$$

$$= \frac{7}{2}(25) - 7(9) + 21 = 42$$
2. The graph of the function \( f(x) \) is pictured below:

(a) Consider integral \( \int_{1}^{4} f(x) \, dx \) with Riemann sums given below. Which values are overestimates for the value of the integral?

\[
R_5, \quad R_{10}, \quad L_5, \quad L_{10}
\]

See that \( f(x) \) is decreasing on \( 1 \leq x \leq 2 \),

**So:** \( L_5, L_{10} \) are overestimates

(b) Consider integral \( \int_{3}^{4} f(x) \, dx \) with Riemann sums given below. Order the values from greatest to least.

\[
R_7, \quad M_7, \quad L_7, \quad T_7
\]

See that \( f(x) \) is increasing and concave up,

**So** we have \( \[ L_7 < M_7 < T_7 < R_7 \] \)

(c) Consider integral \( \int_{2}^{4} f(x) \, dx \) with Riemann sums given below. Which values are underestimates for the value of the integral?

\[
M_7, \quad T_7, \quad M_3, \quad T_3
\]

See that \( f(x) \) is concave up on \( 2 \leq x \leq 4 \),

**So** the midpoint sums are underestimates 

\[
M_7, \quad M_3
\]
3. Define the following area function:

\[ F(x) = \int_0^x \frac{\sin(\sqrt{t})}{3\sqrt{t}} \, dt \]

(a) Find \(F(\pi^2)\).

We'll use substitution to find the antiderivative.

\[ \int \frac{\sin(\sqrt{t})}{3\sqrt{t}} \, dt \]

Let \( u = \sqrt{t} \), then \( du = \frac{1}{2\sqrt{t}} \, dt = \frac{dt}{2\sqrt{t}} \)

So, we have

\[ \int \frac{\sin(\sqrt{t})}{3\sqrt{t}} \, dt = \frac{1}{2} \int \sin(u) \cdot (2\, du) \]

\[ = \frac{2}{3} \int \sin(u) \, du = \frac{-2}{3} \cos(u) + C \]

(b) Find \(F'(\pi^2)\).

See that \( F'(x) = \frac{\sin(\sqrt{x})}{3\sqrt{x}} \)

by FTC 2

So, \( F'(\pi^2) = \frac{\sin(\sqrt{\pi^2})}{3(\sqrt{\pi^2})} \)

\[ = \frac{\sin(\pi)}{3\pi} = 0 \]

Now by FTC 1:

\[ F(\pi^2) = \frac{-2}{3} \cos(\sqrt{\pi^2}) + \frac{2}{3} \cos(\sqrt{0}) \]

\[ = -\frac{2}{3} (-1 + 1) \]

\[ = \frac{4}{3} \]
4. Find the following antiderivative:

\[ \int \frac{x}{2e^{3x}} \, dx \]

Let \( u = x \) and \( dv = \frac{e^{-3x}}{3} \, dx \)

so we have \( du = dx \) and \( v = -\frac{e^{-3x}}{3} \)

Now apply \text{I.B.P}:

\[ \int \frac{x}{2e^{3x}} \, dx = \frac{1}{2} \int \frac{x}{e^{3x}} \, dx = \frac{1}{2} \int udv \]

\[ = \frac{1}{2} \left( uv - \int v \, du \right) \]

\[ = \frac{1}{2} \left( x \left( -\frac{e^{-3x}}{3} \right) - \int -\frac{e^{-3x}}{3} \, dx \right) \]

\[ = \frac{1}{2} \left( -\frac{xe^{-3x}}{3} + \int \frac{e^{-3x}}{3} \, dx \right) \]

\[ = \frac{1}{2} \left( -\frac{xe^{-3x}}{3} + \frac{1}{3}\left( \frac{e^{-3x}}{3} \right) \right) + C \]