Exam rules: No electronic devices of any kind are allowed during this exam. You may use one 2-sided US Letter sized page of notes, but no books or other assistance are allowed during this exam. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Instructions: Write your solutions clearly and legibly and show all of your work. No credit will be given for unsupported or illegible solutions. Please put a box around your answer to each part. If any question is not clear, ask for clarification.
1. (8 points) Suppose that a butterfly has constant acceleration 5 m/s² up and initial velocity 15 m/s down.

(a) If the butterfly starts 10 meters above the top of a tree, find its position function $s(t)$ in meters above the top of the tree at time $t$ seconds after the start.

(b) Find the butterfly’s displacement from time $t = 0$ to $t = 4$ seconds; that is, compute the following integral:

$$\int_0^4 s'(t) \, dt$$
(c) Find the butterfly’s total distance travelled over the same time interval; that is, compute the following integral:

$$\int_{0}^{4} |s'(t)| \, dt$$
2. (6 points) Consider the following function:

\[ F(x) = \int_{3}^{\cos(x)} e^{t^2} \, dt \]

Find the derivative \( F'(x) \).
3. (8 points) Find the following antiderivative:

\[ \int \frac{\ln(x)}{x^5} \, dx \]
4. (8 points) Find the following antiderivative:

\[ \int \sec^5(x) \tan^3(x) \, dx \]
5. (10 points)

(a) Write the partial fraction decomposition for the following rational function:

\[
\frac{3x^2 - 3x + 2}{(x^2 + 1)(x - 3)}
\]
(b) Evaluate its indefinite integral:

\[ \int \frac{3x^2 - 3x + 2}{(x^2 + 1)(x - 3)} \, dx \]
6. (8 points) Determine if the given improper integral converges or diverges. If it converges, find its value.

\[ \int_{0}^{\infty} \frac{2x^2}{(x^3 + 5)^3} \, dx \]
7. (10 points) Consider the following curves and their graphs:

\[ y = 3x - 7 \quad \text{and} \quad y = x^2 - 6x + 11 \]

(a) Setup, but do not evaluate, an integral giving the area of the region enclosed by the two curves.
(b) Setup, but **do not evaluate**, an integral giving the *volume* of the solid obtained by revolving the region in part (a) about the *x*-axis.
8. (8 points) Find the specific solution for the following initial value problem:

\[ \frac{dy}{dx} = 3yx, \quad y(0) = 2 \]
9. (6 points) Consider the following sequence:

\[ a_n = \frac{n^4}{n^2 - n - 1} \]

(a) Determine the limit \( \lim_{n \to \infty} a_n \)

(b) Consider the series:

\[ a_0 + a_1 + a_2 + a_3 + a_4 + \cdots \]

What can you say about its convergence? (Circle one)

Convergent  Divergent  No conclusion

If the series is convergent then give its value.
10. (8 points) Let \( f(x) = \sqrt{x} \). Compute the degree 4 Taylor polynomial approximating \( f(x) \) near \( x = 1 \).