Exam rules:

1. No electronic devices of any kind are allowed during this exam.

2. You may use one 2-sided US Letter sized page of notes, but no books or other assistance are allowed during this exam.

3. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Instructions:

1. Write your first and last name at the top of every page of this exam.

2. Write your solutions clearly and legibly. No credit will be given for illegible solutions.

3. Show the work leading to your solutions. Partial credit can only be given based on the work you show.

4. Please put a box around your final answer to each part.

→ If any question is not clear, ask for clarification.
1. (10 points) Consider the points $A(0, -3, 4)$, $B(7, 1, -1)$, $C(1, 5, -2)$.

(a) Give the vector $\overrightarrow{AC}$ and its length $|\overrightarrow{AC}|$.

(b) Give the vector $\overrightarrow{AB} - \overrightarrow{CB} + 3\overrightarrow{CA}$.

(c) Find the angle $\angle BAC$. (Note: You don’t need to simplify inverse trigonometric functions.)
(d) Find the area of the triangle formed by $A$, $B$, and $C$. 
2. (10 points) Consider the vectors \( \vec{u} = \langle 1, 5, 1 \rangle \) and \( \vec{v} = \langle -3, 2, 2 \rangle \).

(a) Give a parametric (vector) equation for the line perpendicular to both \( \vec{u} \) and \( \vec{v} \) and passing through the point \( (9, 5, -3) \).

(b) Give an equation for the plane containing both the line found in part (a) and the point \( (1, 2, 3) \).
3. (10 points) Consider the plane
\[ \mathcal{P} : -x + 5y - 3z = 9 \]
and the lines
\[ \vec{r}(t) = \langle 2t + 2, 3t - 1, -t \rangle \quad \text{and} \quad \vec{l}(s) = \langle -s + 12, 5s + 14, -5s - 5 \rangle \]
(a) Give the point of intersection of the line \( \vec{r}(t) \) and the plane \( \mathcal{P} \).
(b) Give the point of intersection of the lines $\vec{r}(t)$ and $\vec{l}(s)$. 
4. (10 points) Consider the parameterized curve in \( \mathbb{R}^3 \) given by:

\[
\vec{r}(t) = \langle te^{t^2-1}, t^2e^{t^2-1}, t^3 \ln(t+2) \rangle
\]

Give a parametric (vector) equation for the line tangent to \( \vec{r}(t) \) at the point \( \vec{r}(-1) \).
5. (10 points) Consider the function $f(x, y) = \sin(\pi xy) + xy^2$.

(a) Find the gradient vector for $f(x, y)$ at the point $(1, 1)$. That is, find $\nabla f(1, 1)$.

(b) Find the directional derivative of $f(x, y)$ at the point $(1, 1)$ in the direction of the vector $\langle -3, 5 \rangle$.

(c) Give an equation for the tangent plane in $\mathbb{R}^3$ to the graph $z = f(x, y)$ at the point $(1, 1, f(1, 1))$. 
6. (10 points) Consider the function $f(x, y) = 2x + 4y - x^2 - y^2 - 3$

(a) Find the critical points of $f(x, y)$.

(b) For each critical point, tell whether it is a local minimum, local maximum, or saddle point.
(c) Find the global maximum and global minimum values of $f(x, y)$ over the closed triangular region in $\mathbb{R}^2$ with vertices $(0, 0)$, $(2, 0)$, and $(1, 3)$. 
7. (10 points) Consider the function \( f(x, y, z) = x^2 - y^2 \). Use the method of Lagrange multipliers to determine its maximum and minimum values on the ellipsoidal surface in \( \mathbb{R}^3 \) given by:

\[
x^2 + 2y^2 + 3z^2 = 1
\]
8. (10 points) Consider the following function:

\[ f(x, y) = \frac{1}{(2x + 3y)^2} \]

Find its double integral \( \int \int_R f(x, y) \, dA \) over the rectangle \( R \) given by \( 0 \leq x \leq 1 \) and \( 1 \leq y \leq 2 \).