Math 10C.
Midterm Exam 1
January 29, 2019

Exam rules:

1. No electronic devices of any kind are allowed during this exam.

2. You may use one 2-sided US Letter sized page of notes, but no books or other assistance are allowed during this exam.

3. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Instructions:

1. Write your first and last name at the top of every page of this exam.

2. Write your solutions clearly and legibly. No credit will be given for illegible solutions.

3. Show the work leading to your solutions. Partial credit can only be given based on the work you show.

4. Please put a [box] around your final answer to each part.

→ If any question is not clear, ask for clarification.
1. (25 points total) Consider the following points in $\mathbb{R}^3$:

$A(2, -3, 4) \quad B(-1, 0, 1) \quad C(2, -3, 0)$

(a) (3 points) Give the following vectors in component form: $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\overrightarrow{AB} = \langle -1-2, 0-(-3), 1-4 \rangle$

$\Rightarrow \overrightarrow{AB} = \langle -3, 3, -3 \rangle$

$\overrightarrow{BC} = \langle 2-(-1), -3-0, 0-1 \rangle$

$\Rightarrow \overrightarrow{BC} = \langle 3, -3, -1 \rangle$

(b) (2 points) Find the distance between $A$ and $C$.

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \langle 0, 0, -4 \rangle$

$\text{Distance } (A, C) = |\overrightarrow{AC}| = \sqrt{16} = 4$

(c) (2 points) Give a unit vector (in component form) parallel to the vector $\overrightarrow{AC}$.

$\langle 0, 0, -\frac{4}{4} \rangle = \langle 0, 0, -1 \rangle$
(d) (8 points) Let \( \vec{w} = \overrightarrow{AB} \) and \( \vec{v} = \overrightarrow{BC} \). Find the vector component of \( \vec{w} \) in the direction of \( \vec{v} \). That is, find the projection vector \( \text{proj}_v(\vec{w}) \).

\[
\vec{w} = \langle -3, 3, -3 \rangle \\
\vec{v} = \langle 3, -3, -1 \rangle
\]

\[
\text{proj}_v(\vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|} \left( \frac{\vec{v}}{|\vec{v}|} \right)
\]

\[
= \frac{\langle 3, -3, -1 \rangle \cdot \langle -3, 3, -3 \rangle}{\sqrt{9 + 9 + 1}} \left( \frac{\langle 3, -3, -1 \rangle}{\sqrt{9 + 9 + 1}} \right)
\]

\[
= \frac{-9 + 9 + 3}{\sqrt{19}} \left( \frac{\langle 3, -3, -1 \rangle}{\sqrt{19}} \right)
\]

\[
= \frac{-15}{19} \langle 3, -3, -1 \rangle
\]
(e) (10 points) Find an equation for the plane containing \(A\), \(B\), and \(C\).

\[
\vec{AB} = \langle -3, 3, -3 \rangle \\
\vec{BC} = \langle 3, -3, -1 \rangle
\]

\[\vec{a} = \vec{AB} \times \vec{BC} = \langle -3, 3, -3 \rangle \times \langle 3, -3, -1 \rangle = i \begin{vmatrix} 3 & -3 \\ -3 & -1 \end{vmatrix} + j \begin{vmatrix} -3 & -3 \\ 3 & -1 \end{vmatrix} + k \begin{vmatrix} -3 & 3 \\ 3 & -1 \end{vmatrix} = -9i - 6j - 12k = \langle -12, -12, 0 \rangle
\]

\[\vec{r}_0 = \vec{OA} = \langle 2, -3, 4 \rangle
\]

So the plane is

\[\vec{a} \cdot (\vec{r} - \vec{r}_0) = 0
\]

\(-12(x-2) + 12(y+3) + 0(z-4) = 0
\]

\[\Rightarrow \boxed{\begin{array}{c}
-12(x-2) - 12(y+3) = 0
\end{array}}
\]
2. (15 points total) Consider the following planes in $\mathbb{R}^3$:

$$\mathcal{P}: y - 8z = 3 \quad \text{and} \quad \mathcal{Q}: -x - 2y + 3z = 1$$

(a) (5 points) Find a vector (parametric) equation for the line orthogonal to plane $\mathcal{P}$ and passing through point $B$ from problem 1.

$B(-1, 0, 1)$

$$\vec{v} = \langle 0, 1, -8 \rangle = \text{normal vector of } \mathcal{P}$$

$$\vec{r}_0 = \langle -1, 0, 1 \rangle = \overrightarrow{OB}$$

$$\Rightarrow \quad \vec{r}(t) = \vec{r}_0 + t\vec{v}$$

$$= \langle -1, 0, 1 \rangle + t \langle 0, 1, -8 \rangle$$

$$\Rightarrow \quad \vec{r}(t) = \langle -1, t, 1 - 8t \rangle$$
(b) (10 points) Find a vector (parametric) equation for the line of intersection of the planes $P$ and $Q$.

\[ \vec{n}_P = \langle 0, 1, -8 \rangle \]
\[ \vec{n}_Q = \langle -1, -2, 3 \rangle \]

Then
\[ \vec{v} = \langle 0, 1, -8 \rangle \times \langle -1, -2, 3 \rangle \]
\[ = \begin{vmatrix} 1 & -8 & 0 \\ -2 & 3 & 1 \\ -1 & 3 & 0 \end{vmatrix} + \begin{vmatrix} 1 & -2 & 0 \\ -1 & 3 & 1 \\ 0 & 3 & -1 \end{vmatrix} \]
\[ = \langle 3, -16, -7 \rangle + \langle 0 - 8, 0 + 3, 0 + 2 \rangle \]
\[ = \langle -13, 8, 1 \rangle \]

and a point contained in both planes can be found by:

Let $z = 0$, then
\[ \begin{aligned} y &= 3 \\ -x &= 2y = 1 \end{aligned} \]

So let $T_0 = (5, 3, 0)$ \[ \Rightarrow x = 2y - 1 = 6 - 1 = 5 \]

So:
\[ r(t) = (5, 3, 0) + t \langle -13, 8, 1 \rangle \]