Nothing says ‘I love you’ like a customizable algebraic equation.

By Siobhan Roberts

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Meet Süss, a math widget after your own heart. (You can also visit the widget on its website here, which you might want to do if you’re reading this on a smartphone.)

Like many geometric figures, a heart can be captured in all its curvaceous glory by a single algebraic equation. The equation for a sphere looks simple enough: \( x^2 + y^2 + z^2 = 1 \). A heart is something more complex:

\[
(x^2 + ((1+b)y)^2 + z^2 - 1)^3 - x^2 z^3 - ay^2 z^3 = 0
\]

Süss — German for “sweet” — is an interactive widget that allows you to tweak the algebra and customize the heart to your soul’s delight. It was created for Valentine’s Day by Imaginary, a nonprofit organization in Berlin that designs open-source mathematics programs and exhibitions.

You can stretch and squeeze the heart by moving the two left-most sliders, which change the “a” and “b” parameters; the right-most slider zooms in and out. Better yet, canoodle directly with Süss’s equation and engage in the dialectical interplay between algebra and geometry. (Change that final \( z^3 \) to a \( z^2 \) to see the heart in its underwear.)

In the 17th century the French mathematician and philosopher René Descartes built a bridge between the algebraic and geometric realms when he devised the Cartesian system of coordinates. (He also classified six primitive passions:
wonder and love, hatred and desire, sadness and joy.

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Süss — occupying three-dimensional space, defined by the coordinates x, y and z — also incorporates a more recent concept: “extreme points,” or “singularities,” which are their own subject of study in the field of algebraic geometry.

“If you look at the shape of the heart, you will see one peak on the bottom, and another peak on the top,” said Andreas Daniel Matt, a mathematician and the director at Imaginary. (The peak at the top is indented.)

In general, a singularity typically corresponds to a jump in an otherwise continuous process. In physics, it’s a point with an infinite value. The Big Bang singularity is the biggest singularity in the history of the universe. In general relativity, a singularity is the heart of a black hole from which one would never emerge.

A “naked singularity,” by contrast, would be more forgiving (although it’s not thought to actually exist in nature). “Roughly, a naked singularity is one you could fall into and you could escape from,” said Roger Penrose, a mathematical physicist at the University of Oxford, who has an article on singularity theorems in the forthcoming book “Topology and Physics.”

A singularity is where things go wrong, said Dr. Matt, as also happens in love: “The super high peaks, and the super low peaks. Love is full of extreme points.”

Mathematically, singularities are endowed with special properties, and it’s
these properties that cause problems. A singularity is fragile — it can break with very small changes to the equation; formerly joined surfaces can separate. When singularities break, “they are very difficult to grasp, to study and especially to resolve,” Dr. Matt said.

To see a singularity in action, replace the equation in the Süss widget with a different one: \(x^2+y^2-z^2-a+0.5\). (But use this slightly unsightly version — \(x^2+y^2-z^2-a+0.5\) — so it works in the widget.) Then play with the left-most slider (parameter a), and notice the point where the double cone breaks apart, and where the cones rejoin.

Before Süss was an interactive widget, it was a member of the Gallery of Singularities, a collection of algebraic surfaces created and curated in 2006 by Herwig Hauser, a geometer at the University of Vienna, and a master at resolving singularities.

Other gallery specimens include “Flirt,” “Daisy” — named for Donald Duck’s girlfriend — and “Solitude.” (The word “singularity” comes from the Latin “singularis,” meaning alone.)

Granted, some of the entities in the gallery do not seem so singular — “Sphäre,” for instance. But given the right mathematical techniques, Dr. Hauser said, he could squeeze that sphere down into a point. “Or I could sit on it,” he added.
The process of identifying intriguing algebraic surfaces that possessed singularities sometimes took Dr. Hauser and his students days. More often than not the results were incompatible: attractive equations gave rise to shapes with scant mathematical appeal. Dr. Hauser was matchmaking, essentially.

“Sometimes you play completely on the algebraic side,” he said. “You plug in an equation and discover what the geometric object looks like. And sometimes you do the opposite.”

The opposite — reverse engineering, having a figure in mind and then finding the equation that generates it — is much more difficult, almost impossibly so. Nonetheless, ten years ago Valentina Galata, then a high-school student, saw a poster of the Süss heart during a math-class excursion to an exhibit featuring Surfer, rendering software made by Imaginary that allows users to experiment
in creating algebraic surfaces. Inspired, Galata created a collection of still lifes such as “Cherries” and “Spoon.”

She did not find the task impossible. “Challenging is a better word for it,” said Ms. Galata, now a Ph.D. student in bioinformatics at Saarland University, in Germany. “It was fun to have a tool, using formulas to approximate shapes you have in your head, rather than just drawing on paper.”

Dr. Hauser, a purist, is not fond of rendering real-life objects. In fact, he is not particularly enamored of Süss. Too many people loved it too much: It became the poster heart for algebraic surfaces — on thousands of actual posters in hundreds of Germans schools, and featured in an installation for a shopping-center car park.

“It’s just kitsch,” he said. “Mathematically, it is not so interesting.”

Sebastian Gann, one of Dr. Hauser’s former Ph.D. students on the Gallery of Singularities project, still finds Süss more appealing.

“The intrinsic fascination remains,” he said. “Looking at the image of an emotionally charged symbol, and knowing that the particular shape is defined by some sober maths. How do you perceive it?”

Dr. Hauser prefers “Herz” (German for “heart”), which looks a bit more like an actual human heart:

Herz is alluring for what’s not there, he said: “How do you create a hole in a surface when you choose an equation?”

One of the bigger questions within algebraic geometry is how to “resolve” singularities — that is, how to get rid of them. Mathematically, resolving a singularity means smoothing over the problematic peaks in a surface, but this often requires jumping to a higher dimension.
Alas, in resolving matters of the heart, that strategy seldom works.

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