corrected problems:

§ 3.10 (16) A road perpendicular to a highway leads to a farmhouse located 2 km away.

**Given:**
\[
\frac{s''(t)}{s'(t)} = 80
\]

\[
2 \text{ km} \quad d(t)
\]

\[
s(t)
\]

A car drives past the house at 80 km/h.

How fast is the distance between the car and the house increasing when the car is 6 km past the intersection?

---

See the setup in the above figure.

We are given:

\[
s'(t) = 80 \text{ km/h}
\]

\[
s(0) = 0 \text{ km}
\]

\[
d(0) = 2 \text{ km}
\]
and we are asked to find:

\[ d'(a) \] where \( s(a) = 6 \text{ km} \).

Recall the relation from the Pythagorean theorem:

\[ 2^2 + s(t)^2 = d(t)^2 \]

\[ 2 s(t) s'(t) = 2 d(t) d'(t) \]

\[ \Rightarrow \quad d'(a) = \frac{s(a) s'(a)}{d(a)} \]

So now we need to find the constant on the right hand side:

\[ s(a) = 6 \text{ km}, \text{ by definition} \]
\[ s'(a) = 80 \text{ km/h}, \text{ given} \]
\[ d(a) = \sqrt{4 + s(a)^2} = \sqrt{4 + 36} = \sqrt{40} \text{ km} \]

Therefore:

\[ d'(a) = \frac{(6)(80)}{2\sqrt{10}} \text{ km/h} = \frac{240}{\sqrt{10}} \text{ km/h} \]
Approximate \( \frac{1}{101} \) and compute \( 2\sigma \) error:

Let \( f(x) = \frac{1}{x} \)

Then \( f'(x) = \frac{-1}{x^2} \)

Now we'll use the approximation at \( x=100 \):

\[
L(x) = f'(100)(x-100) + f(100)
\]
\[= \frac{-1}{(100)^2}(x-100) + 0.01
\]
\[= -(0.00001)(x-100) + 0.01
\]
\[= -0.0001x + 0.01 + 0.01 = \frac{-0.0001x + 0.02}{100}
\]

Hence:

\[
\frac{1}{101} = f(101) \approx L(101) = -0.0001(101) + 0.02
\]
\[= -0.0101 + 0.02
\]
\[\approx 0.0099
\]

\[2\sigma - \text{error} = \left| \frac{L(101) - f(101)}{f(101)} \right| \cdot 100
\]
\[= \left| \frac{0.0099 - 0.009990099\cdots}{0.009990099\cdots} \right| \cdot 100
\]
\[\approx 0.00999992
\]
Selected additional problems:

§ 3.9 (20) \( y = 16 \sin x \)

See first that \( y = e^{\ln(16) \sin(x)} \)

so now apply the chain rule:

\[
\frac{dy}{dx} = e^{\ln(16) \sin(x)} \frac{d}{dx} \left( \ln(16) \sin(x) \right)
\]

\[
= 16 \sin(x), \ln(16) \cdot \cos(x)
\]

\[
= \ln(16) \cos(x) \cdot 16 \sin(x)
\]

§ 3.9 (40) \( y = \frac{x(x+1)^3}{(3x-1)^2} \)

First apply \( \ln \):

\[
\ln(y) = \ln \left( \frac{x(x+1)^3}{(3x-1)^2} \right)
\]

\[
= \ln(x(x+1)^3) - \ln((3x-1)^2)
\]

\[
= \ln(x) + 3 \ln(x+1) - 2 \ln(3x-1)
\]

Now apply the chain rule:

\[
\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} + \frac{3}{x+1} - \frac{6}{3x-1}
\]
and thus:

\[ \frac{dy}{dx} = y \left( \frac{1}{x} + \frac{3}{x+1} - \frac{6}{3x-1} \right) \]

\[ = \frac{x(x+1)^2}{(3x-1)^2} \left( \frac{1}{x} + \frac{3}{x+1} - \frac{6}{3x-1} \right) \]

\[ = \frac{(x+1)^3}{(3x-1)^2} + \frac{3x(x+1)^2}{(3x-1)^2} - \frac{6x(x+1)^3}{(3x-1)^3} \]

§ 3.10 (25) A police car traveling south toward Sioux Falls at 160 km/h pursues a truck traveling east from Sioux Falls at 140 km/h.

At time \( t=0 \), the police car is 20 km north and the truck is 30 km east. (See figure 1)
Calculate the rate at which the distance between vehicles is changing

(a) at \( t = 0 \)

(b) at \( t = 5 \) minutes

Consider the diagram

\[ p(t) \]

\[ q(t) \]

\[ d(t) \]

\( p(t) = \) distance of police from Sioux Falls
\( q(t) = \) distance of truck from Sioux Falls
\( d(t) = \) distance between them

Now we have:

Given:

\[ p(0) = 20 \text{ km} \quad p'(t) = -160 \text{ km/h} \]
\[ q(0) = 30 \text{ km} \quad q'(t) = 140 \text{ km/h} \]

Want to find:

\[ d'(0) \]

\[ d''(5 \text{ minutes}) \]
First, since \( p \) and \( q \) have constant derivatives, we may use the tangent line to describe them:

\[
p(t) = -160t + 20 \\
q(t) = 170t + 30
\]

And since \( p, q, d \) must satisfy the Pythagorean relation:

\[
p(t)^2 + q(t)^2 = d(t)^2
\]

we can write: (by chain rule)

\[
2d(t)d'(t) = 2p(t)p'(t) + 2q(t)q'(t)
\]

\[
\Rightarrow d'(t) = \frac{p(t)p'(t) + q(t)q'(t)}{d(t)}
\]

and

\[
d(t) = \sqrt{p(t)^2 + q(t)^2}
\]
So now:

\[ d'(0) = \frac{p(0)p'(0) + q(0)q'(0)}{d(0)} \]

\[ = \frac{(20 \text{ km})(-160 \text{ km/h}) + (30 \text{ km})(140 \text{ km/h})}{\sqrt{(20 \text{ km})^2 + (30 \text{ km})^2}} \]

\[ = \frac{-320 + 420}{\sqrt{400 + 900}} = \frac{100}{\sqrt{1300}} \]

\[ = \frac{10}{\sqrt{13}} \text{ km/h} \]

\[ d'(5 \text{ min.}) = d'(\frac{5}{60} \text{ hour}) = d'(\frac{1}{12}) \]

\[ = \frac{p(\frac{1}{12})p'(\frac{1}{12}) + q(\frac{1}{12})q'(\frac{1}{12})}{d(\frac{1}{12})} \]

\[ = \frac{(\frac{160}{12} + 20)(-160) + (\frac{140}{12} + 30)(140)}{\sqrt{(\frac{-160}{12} + 20)^2 + (\frac{140}{12} + 30)^2}} \]

\[ = \frac{(\frac{20}{3})(-160) + (\frac{125}{3})(140)}{\sqrt{\frac{400}{9} + \frac{125^2}{9}}} = \frac{-14300}{3 \sqrt{16025}} = \frac{14300}{\sqrt{16025}} \text{ km/h} \]
§3.10 26 A car travels down a highway at 25 m/s. An observer stands 150 m from the highway. (Figure 1)

(a) How fast is the distance from the observer to the car increasing when the car passes in front of the observer? (Explain u/o calculation)

(b) How fast is the distance increasing 20 s later?

![Diagram](image)
Given:

\[ d(0) = 150 \text{ m} \]
\[ s(0) = 0 \text{ m} \quad s'(t) = 25 \text{ m/s} \]

(a) Want: \( d'(0) = ? \)

Notice that \( d(t) \) is decreasing for \( t < 0 \) and that \( d(t) \) is increasing for \( t > 0 \).

Since \( d \) is differentiable at \( t = 0 \), this tells us that \( d'(0) = 0 \).

(try graphing \( d(t) \) to see why!)

(b) Want: \( d'(20) \)

First, since \( s'(t) \) is constant, we have \( s(t) = 25t \).
Now apply $\frac{dt}{dt}$ to the Pythagorean relation:

\[ 150^2 + s(t)^2 = d(t)^2 \]

\[ \Rightarrow 2s(t)s'(t) = 2d(t)\frac{d'(t)}{d(t)} \]

\[ \Rightarrow d'(20) = \frac{s(20)s'(20)}{d(20)} \]

\[ = \frac{25(20)\cdot (25)}{\sqrt{150^2 + s(20)^2}} \]

\[ = \frac{12500}{\sqrt{22500 + 250000}} \]

\[ = \frac{12500}{\sqrt{272500}} \approx \frac{1250}{\sqrt{272}5} \text{ km/h} \]
A particle moves counter-clockwise around ellipse \( 9x^2 + 16y^2 = 25 \)

(a) In which of the quadrants is \( \frac{dx}{dt} > 0 \)?

Clearly the \( x \)-coordinate (i.e., the function \( x(t) \)) is **increasing** in quadrants III and IV, hence \( \frac{dx}{dt} > 0 \) in III and IV.

(b) Find a relation between \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \)?

Apply \( \frac{d}{dt} \) to \( 9x^2 + 16y^2 = 25 \):

\[ 18x \frac{dx}{dt} + 32y \frac{dy}{dt} = 0 \]

\[ \Rightarrow \frac{dy}{dt} = -\frac{9}{16} \cdot \frac{dx}{dt} \]
(c) At what rate is the $x$-coord. changing when the particle passes (1,1)? If the $y$-coord is changing at $6$ m/s?

\[
\begin{align*}
\text{Given:} & \quad x(0) = 1 \text{ m} \\
& \quad y(0) = 1 \text{ m} \\
& \quad y'(0) = 6 \text{ m/s} \\
\text{Want:} & \quad x'(0) = ?
\end{align*}
\]

So now using part (b):

\[
18(1)x'(0) + 32(1)(6) = 0
\]

\[
\Rightarrow x'(0) = \frac{-6 \cdot 32}{18} = \frac{-32}{3} \text{ m/s}
\]

(d) What is $\frac{dy}{dt}$ at the top and bottom?

\[
\frac{dy}{dt} = 0 \quad \text{for} \quad (x, y) = (0, 1) \quad \text{(top)}
\]

and

\[
(x, y) = (0, -1) \quad \text{(bottom)}
\]

(What is $\frac{dy}{dt}$ just before and after these points?)
§4.1 20. Estimate \( \frac{1}{\sqrt{98}} - \frac{1}{10} \)

Let \( f(x) = \frac{1}{\sqrt{x}} \) and we'll estimate with the linear approx. at \( x = 100 \)

So: \( f'(x) = -\frac{1}{2} x^{-3/2} = -\frac{1}{2(\sqrt{x})^3} \)

and then:

\[
f'(100) = -\frac{1}{2} (100)^{-3/2} = -\frac{1}{2} (10)^{-3} = \frac{-1}{2000}
\]

And the linear approximation is:

\[
L(x) = \frac{-1}{2000} (x - 100) + f(100)
\]

\[
= \frac{-x}{2000} + \frac{1}{20} + \frac{1}{10}
\]

So now

\[
\frac{1}{\sqrt{98}} - \frac{1}{10} \approx L(98) - \frac{1}{10} = \frac{-98}{2000} + \frac{1}{20}
\]
§4.1 (54) Find linear approx. of \( y = e^x \ln(x) \) at \( a = 1 \)

\[
y'(x) = e^x \frac{d}{dx} (\ln(x)) + e^x \ln(x) = \frac{e^x}{x} + e^x \ln(x)
\]

\[
y'(1) = \frac{e^1}{1} + e^1 \ln(1) = e
\]

\[
y(1) = e^1 \ln(1) = 0
\]

So then:

\[
L(x) = e(x-1) + 0 = \boxed{e^x - e}
\]

§4.1 (59) Approximate \( \sqrt[3]{17} \) and find % error

Using §4.1 (20) (see page 14) we have (for \( f(x) = \sqrt{x} \))

\[
f'(x) = \frac{-1}{2(\sqrt{x})^3}
\]

we'll use \( L(x) \) at \( a = 16 \).

Then

\[
L(x) = f'(16) (x-16) + f(16) = \frac{-1}{128} (x-16) + \frac{1}{4}
\]

So now

\[
\frac{1}{\sqrt[3]{17}} \approx L(17) = \frac{-1}{128} + \frac{1}{4} \approx 0.2421875 \quad \text{error} = 1.435\%.
\]