48. \( \frac{dy}{dt} = 3 - 2t \), \( y(0) = -5 \)

First, the general antiderivative is:

\[
y(t) = 3t - \frac{2t^2}{2} + C
\]

\[
= 3t - t^2 + C
\]

and the specific antiderivative has:

\[-5 = y(0) = 3(0) - (0)^2 + C
\]

\[
= C
\]

So,

\[
y(t) = 3t - t^2 - 5
\]
First, the general antiderivative is:

\[ z(t) = \frac{t^{-1/2}}{-1/2} + C \]

\[ = -2t^{-1/2} + C \]

and the specific antiderivative satisfies:

\[ -1 = z(4) = -2(4)^{-1/2} + C \]

\[ \Rightarrow -1 + \frac{2}{\sqrt{4}} \neq C \]

\[ \Rightarrow -1 + 1 = C \]

\[ \Rightarrow 0 = C \]

hence \[ z(t) = -2t^{-1/2} \]
Selected additional solution:

56. \( \frac{dy}{dz} = \sin(2z), \ y\left(\frac{\pi}{4}\right) = 4 \)

General: \( y(z) = -\cos(2z) + C \)

Specific:

\[ 4 = y\left(\frac{\pi}{4}\right) = -\frac{\cos\left(\frac{\pi}{2}\right)}{2} + C \]

\[ \Rightarrow \ 4 = 0 + C \Rightarrow C = 4 \]

Therefore:

\[ y(z) = -\frac{\cos(2z)}{2} + 4 \]
\[ \frac{dy}{dt} = e^{-t}, \quad y(0) = 0 \]

**General:**

\[ y(t) = -e^{-t} + C \]

**Specific:**

\[ 0 = y(0) = -e^{-0} + C = -1 + C \]

\[ \Rightarrow C = 1 \]

Hence,

\[ y(t) = -e^{-t} + 1 \]
\[ f''(x) = x^3 - 2x + 1 \]
\[ f'(1) = 0 \quad (1) \]
\[ f(1) = 4 \quad (2) \]

Find general antiderivative

\[ f'(x) = \frac{x^4}{4} - x^2 + x + C \]

Find specific antiderivative

\[ 0 = f'(1) = \frac{(1)^4}{4} - (1)^2 + 1 + C \]
\[ \frac{0}{4} = \frac{1}{4} + C \quad \rightarrow \quad C = -\frac{1}{4} \]

Hence:

\[ f'(x) = \frac{x^4}{4} - x^2 + x - \frac{1}{4} \]

Second general antiderivative

\[ f(x) = \frac{x^5}{5 \cdot 4} - \frac{x^3}{3} + \frac{x^2}{2} - \frac{1}{4} x + C \]

Second specific antiderivative:

\[ 4 = f(1) = \frac{(1)^5}{5 \cdot 4} - \frac{(1)^3}{3} + \frac{(1)^2}{2} - \frac{(1)}{4} + C \]
\[ (2) \]
hence:

\[
4 = \frac{1}{20} - \frac{1}{3} + \frac{1}{2} + \frac{1}{4} + C
\]

\[
4 - \frac{11}{20} + \frac{7}{12} = C
\]

So:

\[
f(x) = \frac{x^5}{20} - \frac{x^3}{3} + \frac{x^2}{2} - \frac{x}{4} + \left(4 - \frac{11}{20} + \frac{7}{12}\right)
\]